Maximizing Recommender's Influence in a Social Network: An Information Theoretic Perspective

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***Abstract*—We study a social network in which individuals make decisions influenced by a recommender as well as the previous actions taken by themselves or other users. The recommender aims to tailor its suggestions to maximize the benefit from utilizing social interactions. We refer to this benefit as the recommender's *influence* which, in essence, measures the value of controlling the specific suggestions offered to the individuals. We show that this influence can be quantified by the *directed information* between the suggestions and people's actions. Accordingly, we identify the precise relationship between the social network-based recommendation system and a finite state communication channel whose capacity analysis provides the solution for the influence maximization problem for the recommender. Our results demonstrate that a recommender that tailors its suggestions based on the social dynamics of its customer base can have a significantly greater influence.**

I. Introduction

Social networks have become powerful tools for connecting virtual communities and exchanging information. The impact of mutual relationships in social behavior has been the focus of extensive research, ranging from spread of information [1] and recommender software design [2], to selecting key users to maximize market profit or influence [3]. Interpersonal influence and social advertising have also been acknowledged as major marketing strategies [4], [5]. As such, online systems are now integrating friend interactions into their recommendation process, by allowing users to link to their social media accounts and view the preferences of their friends.

Information-theoretic approaches to social and recommendation networks to date are limited. Entropy is utilized in [6] to classify user activity on Twitter and to categorize the contents, whereas transfer entropy is incorporated in [7] to identify influential users and hidden network structures. The timing information of tweets is utilized in [8] to quantify the influence between two users on Twitter. In [9], belief propagation is applied to evaluate marginal probabilities of user ratings for recommender systems. The error performance for estimating an underlying user rating matrix is analyzed in [10].

Informational relationships in complex systems have first been characterized in [11] to provide a quantitative description of directional information flow. Subsequently, directed information is introduced to study channels with causal feedback in [12], [13]. It has found various applications in hypothesis testing, instantaneous data compression, and portfolio theory [14], where it is used to quantify the increase in growth rate in a gambling scenario with causal side information. Directed information graphs are introduced in [15].

In this paper, we model the interaction between a recommender and an individual user as a *recommendation channel*. Using this base model, we consider a social network with interacting users and a central recommender that wishes to influence the network. An example of this setting is a video streaming website that makes personalized recommendations to its subscribers. The subscribers can share their opinions on social media, thereby affecting the actions of others. In our model, we want to reflect the influence of friendship, dominance, or credibility on decision patterns by allowing an individual to be influenced by other users and previous decisions as well as the recommender. As such, our social model inherently has *causal feedback* of past decisions at the users and the recommender, often available in practice in the form of a reply message, retweet or a reshare.

In our setting, a recommender controlling the suggestions has a competitive advantage in estimating the decisions of the social network. We quantify this advantage as done for a gambler with side information in [16], and adopt it as a measure of the recommender's *influence* on the network. We then solve the problem of maximizing this influence by choosing the optimal recommendation strategy. In particular, the recommender is to choose each suggestion based on causal feedback such that it knows more about the network than an observer without access to the suggestions. We show that this influence is quantified by *directed information*, and that the influence maximization problem is equivalent to finding the capacity of a finite state feedback channel. Our main contributions can be summarized as follows:

- We provide an information-theoretic model for an interactive recommendation network.
- We quantify the influence of the recommender on the social network using directed information.
- We determine the fundamental limits of the recommender's influence, and corresponding recommendation strategies.

We anticipate this study to facilitate research directions that integrate social networks and information theory.

II. System Model

A. Base Model: Recommendation Channel

We initially consider an interaction process between an individual user and a recommender given in Fig. 1. The set of possible recommendations are given as $\mathcal{K} = \{1, 2, \dots, K\}$,

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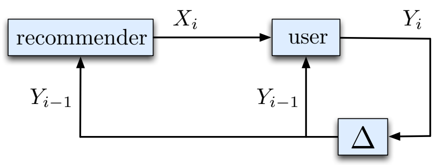

Fig. 1. Recommendation channel.

where each suggestion may represent buying a product, voting for a candidate, and so on. Starting from $i = 1$, the recommender makes a suggestion $X_i \in \mathcal{K}$ to the user, for $i = 1, \ldots, n$. In response to each suggestion, the user makes a decision $Y_i \in \mathcal{Y} = \mathcal{K} \cup \{e\}$, where e denotes not taking any action, i.e., indifference. The decision at time i is influenced by both the recommender's suggestion X_i and the user's previous decision, Y_{i-1}. The recommender has causal knowledge of all previous decisions $Y^{i-1} = (Y_1, \ldots, Y_{i-1})$ when choosing X_i.

The decision at time i is governed by the probability $p(Y_i|X_i, Y_{i-1})$, which denotes the probability of choosing Y_i given the recommender's suggestion X_i and the previous decision Y_{i-1}. We parameterize this probability as shown in Fig. 2, where an edge between node x_i and node y_i represents the probability $p(Y_i = y_i|X_i = x_i, Y_{i-1} = y_{i-1})$. For example, the probability of accepting a suggestion $X_i = k$ that is identical to the previous decision $Y_{i-1} = k$ is

$$p(Y_i = k|X_i = k, Y_{i-1} = k) = \alpha, \quad (1)$$

and the probability of not taking an action given $Y_{i-1} = e$ is

$$p(Y_i = e|X_i = k, Y_{i-1} = e) = \rho. \quad (2)$$

The remaining parameters are found by uniformly distributing the remaining probability among options, e.g., for $j \neq k$,

$$p(Y_i = j|X_i = k, Y_{i-1} = k) = \frac{1 - (\alpha + \phi)}{K - 1}. \quad (3)$$

Fig. 2a characterizes the bias towards the previous action, whereas Fig. 2b applies when the person has not taken an action at the previous time instant. We choose $Y_0 = e$ to avoid imposing a bias on the decisions of the person. To ensure valid probability distributions, the parameters in Fig. 2 satisfy

$$\text{i)}\quad 0 < \alpha, \phi < 1, \qquad \alpha + \phi < 1, \quad (4)$$
$$\text{ii)}\quad 0 < \beta, \theta, \mu < 1, \qquad \beta + \theta + \mu < 1, \quad (5)$$
$$\text{iii)}\quad 0 < \gamma, \rho < 1, \qquad \gamma + \rho < 1. \quad (6)$$

We view the recommendation process in Fig. 1 with the behavior dynamics in Fig. 2 as a finite state channel with causal feedback. That is, when determining the suggestion X_i at time instant i, the recommender has access to the previous decisions, i.e., Y^{i-1}. Since the recommender can observe the previous decisions in the network, this channel exhibits causal feedback. The corresponding channel is characterized by the probability $p(Y_i|X_i, S_i)$, where the *state* S_i of the channel is the previous decision Y_{i-1}. We refer to this finite state channel as the *recommender channel*. This equivalence allows us to study the fundamental limits of the recommender channel from the perspective of point-to-point finite state channels.

B. Model Extensions

The recommendation process from Section II-A can be immediately generalized to the case when the user is influenced

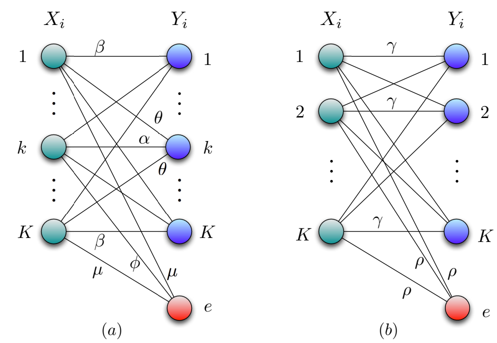

Fig. 2. Decision probabilities with suggestions $X_i \in \mathcal{K}$ and decisions $Y_i \in \mathcal{Y}$, for the cases (a) $Y_{i-1} = k \in \mathcal{K}$ and (b) $Y_{i-1} = e$.

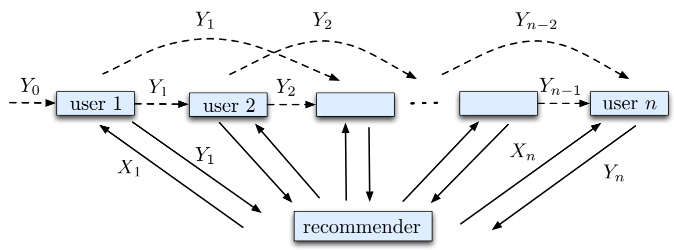

Fig. 3. Recommendation system with $M = 2$. Dashed lines represent the social influence between users whereas solid lines represent the forward and feedback channels between a user and the recommender.

not only by its most recent action, but by the past $M \in \mathbb{Z}^+$ actions. To do so, we define the channel output as a vector

$$\mathbf{Y}_i = (Y_i, Y_{i-1}, \ldots, Y_{i-(M-1)}) \quad (7)$$

which also includes the past M decisions. We characterize the corresponding channel by the probability $p(\mathbf{Y}_i|X_i, \mathbf{Y}_{i-1})$ and for every $\mathbf{v} = (v_1, \ldots, v_M) \in \mathcal{Y}^M$, $\mathbf{u} = (u_1, \ldots, u_M) \in \mathcal{Y}^M$,

$$p(\mathbf{Y}_i = \mathbf{v}|X_i, \mathbf{Y}_{i-1} = \mathbf{u}) = p(Y_i = v_1|X_i, \mathbf{Y}_{i-1} = \mathbf{u}) \quad (8)$$

if $v_{t+1} = u_t$ for $t = 1, \ldots, M-1$, and $p(\mathbf{Y}_i = \mathbf{v}|X_i, \mathbf{Y}_{i-1} = \mathbf{u}) = 0$ otherwise.

Our model can also be utilized to study a network with n users and a recommender as in Fig. 3. In this model, at time instant i, the recommender makes a suggestion $X_i \in \mathcal{K}$ to user i. User i makes a decision $Y_i \in \mathcal{Y}$, influenced by both the recommender's suggestion X_i and the decision of the past M users $(Y_{i-1}, \ldots, Y_M)$. Then, $p(\mathbf{Y}_i|X_i, \mathbf{Y}_{i-1})$ denotes the probability of $\mathbf{Y}_i$ given X_i and the observed decisions of the previous users $\mathbf{Y}_{i-1}$. The social network in Fig. 3 can also be transformed into a virtual finite state channel with causal feedback. Therefore, the following results apply to both social scenarios. For simplicity, we assume $M = 1$ in the sequel.

III. Influence Maximization for the Recommender

In this section, we quantify a measure for the influence of the recommender over others in the social network, and solve the problem of influence maximization. We first relate the influence of the recommender to the increase in growth rate for the gambling problem in [14]. We consider the decisions of users as random events, where betting on the correct outcome generates profits. For example, buying the stocks of a company is a bet played on the outcome that an individual, or collectively the network, will choose the product of said company over its competitors, therefore yielding a revenue for its shareholders. In this setting, knowing the suggestions made to each individual clearly gives the recommender an advantage when choosing which companies to invest in.

From the perspective of an outsider that is unaware of the suggestions X^n delivered to each user, the sequence of decisions Y^n has the probability distribution $p(y^n)$. The optimal investment strategy that maximizes the growth rate of the outsider's wealth is proportional to the payoffs [14], [16], [17], yielding the growth rate

$$W(Y^n) = \sum_{y^n} p(y^n) \log o(y^n) - H(Y^n), \tag{9}$$

where $o(y^n)$ is the payoff for unit investment in the outcome y^n. On the other hand, knowing suggestions X^n, the recommender's optimal causal gambling strategy yields the growth rate [14, Thm. 1]

$$W(Y^n||X^n) = \sum_{y^n} p(y^n) \log o(y^n) - H(Y^n||X^n), \tag{10}$$

where $H(Y^n||X^n)$ is the causally conditional entropy [13]

$$H(Y^n||X^n) = \sum_{i=1}^{n} H(Y_i|Y^{i-1}, X^i). \tag{11}$$

The increase in growth rate with the knowledge of X^n, which we refer to as the *influence* of the recommender, is therefore

$$\begin{aligned} \Delta W(Y^n||X^n) &= W(Y^n||X^n) - W(Y^n) && (12)\\ &= H(Y^n) - H(Y^n||X^n) && (13)\\ &= I(X^n \to Y^n), && (14) \end{aligned}$$

where (14) is the directed information [12], defined as

$$I(X^n \to Y^n) = \sum_{i=1}^{n} I(X^i; Y_i|Y^{i-1}). \tag{15}$$

Note that this is in agreement with [14, Corollary 1]. We consider the case where the recommender chooses the suggestions based on a causally conditioned input distribution [13],

$$p(x^n||y^n) = \prod_{i=1}^{n} p(x_i|x^{i-1}, y^{i-1}), \tag{16}$$

i.e., the recommender chooses x_i with probability $p(x_i|x^{i-1}, y^{i-1})$ for $i = 1, \ldots, n$. In this case, the recommender can design the input distribution in (16) to maximize its influence, by solving the influence maximization problem

$$\lim_{n\to\infty} \max_{p(x^n||y^n)} \frac{1}{n} I(X^n \to Y^n). \tag{17}$$

Note that (17) is identical to the problem of finding the capacity of the recommender channel, which is a finite state channel with causal feedback [18]. In fact, feedback capacity can be interpreted as the maximum rate of information flow from the recommender to the actions. In the sequel, we focus on the capacity of the recommender channel in order to find the influence-maximizing strategy for the recommender.

When feedback is equal to the previous output, (17) can be attained within a class of probability mass functions satisfying

$$p(x_i|x^{i-1}, y^{i-1}) = p(x_i|y_{i-1}), \quad i = 1, \ldots, n, \tag{18}$$

as shown in [19], [20]. We then denote

$$p^*(x^n||y^n) = \prod_{i=1}^{n} p(x_i|y_{i-1}). \tag{19}$$

Due to the distribution in (19), the output satisfies the Markov chain $Y^{i-2} - Y_{i-1} - Y_i$ [19]. Moreover, by construction of the channel, we have the Markov chain $X^{i-1} - Y_{i-1} - Y_i$. As a result, the directed information in (15) can be written as

$$\begin{aligned} I(X^n \to Y^n) &= \sum_{i=1}^{n} \left(H(Y_i|Y^{i-1}) - H(Y_i|X^i, Y^{i-1})\right) && (20)\\ &= \sum_{i=1}^{n} \left(H(Y_i|Y_{i-1}) - H(Y_i|X_i, Y_{i-1})\right) && (21)\\ &= \sum_{i=1}^{n} I(Y_i; X_i|Y_{i-1}). && (22) \end{aligned}$$

Hence the feedback capacity becomes

$$C = \lim_{n\to\infty} \max_{p^*(x^n||y^n)} \frac{1}{n} \sum_{i=1}^{n} I(Y_i; X_i|Y_{i-1}), \tag{23}$$

where $p^*(x^n||y^n)$ is defined in (19). Note that the terms on the right side of (19) can be different for each user index i.

We explain now that the solution to (23) is a stationary distribution, i.e., $p(x_i|y_{i-1}) = p(x_j|y_{j-1})$, $i, j \in \{1, 2, \ldots, n\}$, by using the conditions in [18]. Let a directed *edge* exist from state k to state j if

$$\min_{m\in\mathcal{K}} \{p(Y_i = j|X_i = m, Y_{i-1} = k)\} > 0. \tag{24}$$

It can be observed from Fig. 2 that a directed *path* exists from k to j for all $j, k \in \mathcal{Y}$, hence the Markov chain is strongly irreducible. In addition, every state has a self loop, hence their period is one, and the Markov chain is strongly aperiodic.

Next, we provide a set of sufficient conditions on network parameters that ensures the state transition matrix is full column rank.

Theorem 1. A recommender can maximize its influence in the social network by a stationary recommendation distribution if

$$\begin{aligned} &\text{i)} \quad \alpha > \theta && (25)\\ &\text{ii)} \quad \mu \geq \phi && (26)\\ &\text{iii)} \quad \beta > \left(\overline{\theta + \mu}\right)/(K-1). && (27) \end{aligned}$$

Proof: This result immediately follows from constructing the state transition matrix for $k \in \mathcal{Y}$ and applying Gaussian elimination. One then observes that the matrix has either full rank or is of rank 1. The former case readily satisfies the conditions in [18]. For the latter, we refer to [20] for the details on how the conditions in [18] are still satisfied. ■

Remark 1. In Thm. 1, (25) implies that a person is more likely to accept a recommendation when the corresponding action is also taken before. If the recommendation is different from the previous action, accepting the recommendation is arguably less probable than the previous case. The condition in (26) implies that a person is less likely to ignore a recommendation if the same action is taken previously. Finally, (27) is equivalent to $\beta > \overline{\frac{\beta+\theta+\mu}{K-2}}$, which states that if the recommendation is different from the previous action, a person is more likely to accept the recommendation than to choose an action neither recommended nor taken previously. We remark that these conditions primarily reflect the tendency of a person to prefer an action that was also previously taken. Hence, the conditions in Thm. 1 are consistent with the nature of *social marketing*.

Let $p(x|y)$ denote the stationary distribution of channel input $x_i \in \mathcal{K}$ when the channel state is $y_{i-1} \in \mathcal{Y}$, i.e., $p(x_i|y_{i-1}) = p(x|y)$ for all $x = x_i$, $y = y_i$, and $i = 1, \ldots, n$. Then, (23) can be rewritten as

$$C = \max_{p(x|y)} \sum_{k\in\mathcal{Y}} \pi_k I(X_i; Y_i|Y_{i-1} = k), \tag{28}$$

where

$$\pi_k = \lim_{i\to\infty} p(Y_i = k) \tag{29}$$

is the stationary distribution of the channel output process induced by the stationary input distribution $p(x|y), x\in\mathcal{K}, y \in \mathcal{Y}$.

In order to determine the capacity of the recommender channel, we first evaluate the stationary Markov distributions. We denote the conditional distribution of the recommender's input to user i given the decision from user $i-1$ as

$$p(X_i = k|Y_{i-1} = k) = \lambda, \quad \forall k \in \mathcal{K}, \tag{30}$$

which is equal for all k due to the symmetry of the channel for $k = 1, \ldots, K$. In the sequel, we denote

$$\overline{a_1 + \ldots + a_i} = 1 - (a_1 + \ldots + a_i) \tag{31}$$

for $a_1, \ldots, a_i \in \mathbb{R}$ and $i \in \mathbb{N}$. Since $\sum_{j=1}^{K} p(X_i = j|Y_{i-1} = k) = 1$, due to symmetry, we have

$$p(X_i{=}j|Y_{i-1}{=}k) = \frac{\bar{\lambda}}{K-1}, \;\; j \neq k, \;\; \forall j \in \mathcal{K}. \tag{32}$$

We choose the conditional input probabilities for $Y_{i-1} = e$ as

$$p(X_i = k|Y_{i-1} = e) = \frac{1}{K}, \quad \forall k \in \mathcal{K} \tag{33}$$

again due to the symmetry of the channel. As a result, the transition probabilities for the induced Markov chain are

$$p(Y_i = e|Y_{i-1} = k) = \phi\lambda + \mu\bar{\lambda}, \tag{34}$$
$$p(Y_i = k|Y_{i-1} = k) = \alpha\lambda + \theta\bar{\lambda}, \tag{35}$$
$$p(Y_i = j|Y_{i-1} = k) = \frac{1 - \lambda(\alpha + \phi) - \bar{\lambda}(\theta + \mu)}{K-1}, \tag{36}$$
$$p(Y_i = k|Y_{i-1} = e) = \frac{1-\rho}{K}, \tag{37}$$
$$p(Y_i = e|Y_{i-1} = e) = \rho, \tag{38}$$

for $j, k \in \mathcal{K}$ and $j \neq k$. The stationary output distributions can be determined from the global balance equations

$$\pi_k \sum_{j\in\mathcal{Y}} p(Y_i = j|Y_{i-1}{=}k) = \sum_{j\in\mathcal{Y}} \pi_j p(Y_i{=}k|Y_{i-1}{=}j) \tag{39}$$

for $k \in \mathcal{Y}$. Due to symmetry, we have $\pi_1 = \pi_2 = \ldots = \pi_K$. Substituting this and (34)-(38) in (39) yields the solution

$$\pi_e = \frac{\phi\lambda + \mu\bar{\lambda}}{1 - \rho + \phi\lambda + \mu\bar{\lambda}}, \tag{40}$$
$$\pi_k = \frac{1-\rho}{K(1 - \rho + \phi\lambda + \mu\bar{\lambda})}, \quad k = 1, \ldots, K. \tag{41}$$

We are now ready to present our main result:

Theorem 2. The capacity of a recommender channel with $K \geq 2$ recommendations, that satisfies Thm. 1, is given by (42), where we have defined $H(\varphi_1, \varphi_2, \ldots \varphi_i) = -\sum_{j=1}^{i} \varphi_j \log \varphi_j$.

Proof: We first rewrite (28) using (30), (32) as

$$C = \max_{p(x_i|y_{i-1})} \sum_{k\in\mathcal{Y}} I(Y_i; X_i|Y_{i-1}) \tag{43}$$
$$= \max_{\lambda\in[0,1]} \sum_{k\in\mathcal{Y}} I(Y_i; X_i|Y_{i-1}) \tag{44}$$
$$= \max_{\lambda\in[0,1]} \sum_{k\in\mathcal{Y}} \pi_k (H(Y_i|Y_{i-1}{=}k) - H(Y_i|X_i, Y_{i-1} = k)) \tag{45}$$

The expression in (42) is then obtained by evaluating the entropy values from model parameters, and substituting (40)-(41) in (45). ■

Remark 2. For a binary recommendation network, i.e., $K = 2$, the channel parameters satisfy $\beta + \theta + \mu = 1$ by construction, which implies $\overline{\beta + \theta + \mu} = 0$. In this case, we drop the $\bar{\lambda}(\overline{\beta + \theta + \mu}) \log(K-2)$ term in (42) to obtain the capacity.

IV. Numerical Results

We evaluate the influence of the recommender in various social scenarios with different channel parameters. We first analyze the influence of a *trusted* recommender in comparison to an *untrusted* recommender. We say that a recommender is trusted if users follow the recommender's suggestions with relatively high probability. We expect a trustworthy recommender to obtain a greater influence, since it would be more successful at getting its recommendations accepted. The trustworthiness of a recommender is reflected in the parameters β, α and γ in Fig. 2. Trustworthiness increases with β, α and γ. Recall that α is the probability of accepting a recommendation that is the same as the previous decision, whereas β is that of accepting the recommendation despite taking a different action previously. Thus, a natural choice would be to let $\beta < \alpha$. On the other hand, γ refers to the case when no action has been taken previously, hence the user is not affected by the previous decision when reacting. Based on these definitions, for some β, we let $\gamma = \beta + 0.1$ and $\alpha = \beta + 0.3$, and refer to β as the *trustworthiness* of the recommender, which is an overall indicator of the recommender's reputation. The

$$C = \max_{\lambda\in[0,1]} \frac{\bar{\rho}}{\bar{\rho} + \phi\lambda + \mu\bar{\lambda}} \Bigg(H(\alpha\lambda + \theta\bar{\lambda}, \phi\lambda + \mu\bar{\lambda}, \overline{\lambda(\alpha + \phi) + \bar{\lambda}(\theta + \mu)}) - \lambda H(\alpha, \phi, \overline{\alpha + \phi}) - \bar{\lambda} H(\theta, \mu, \beta, \overline{\beta + \theta + \mu})$$
$$+ \bar{\lambda}(\overline{\theta{+}\mu}) \log(K{-}1) - \bar{\lambda}(\overline{\beta{+}\theta{+}\mu}) \log(K{-}2) \Bigg) + \frac{\phi\lambda{+}\mu\bar{\lambda}}{\bar{\rho}{+}\phi\lambda{+}\mu\bar{\lambda}} \Bigg(H(\rho, \bar{\rho}) - H(\gamma, \rho, \overline{\gamma{+}\rho}) + \bar{\rho} \log K - (\overline{\gamma{+}\rho}) \log(K{-}1) \Bigg) \tag{42}$$

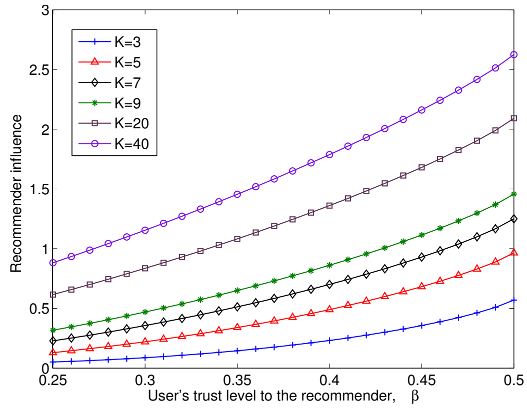

Fig. 4. Recommender influence vs. trustworthiness of the recommender for various number of possible suggestions K.

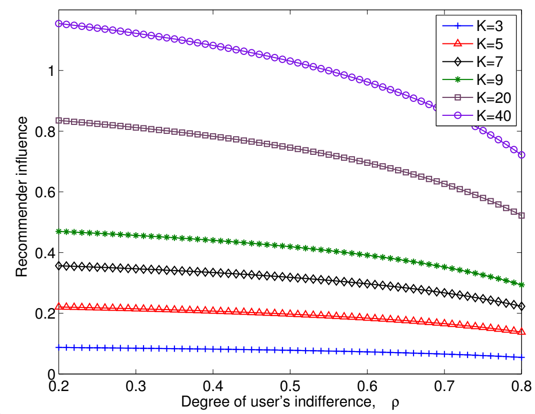

Fig. 5. Recommender influence vs. indifference for various number of possible suggestions.

remainder of the parameters are chosen as $\theta = 0.3$, $\mu = 0.2$, $\phi = 0.2$, $\rho = 0.2$, in accordance with the conditions in (25)-(27) in Thm. 1. The maximum influence the recommender can achieve is illustrated in Fig. 4 for $0.25 \leq \beta \leq 0.5$. We observe that the influence of the recommender significantly increases as its trustworthiness β increases. It can also be seen that the influence of the recommender increases with the number of actions K. This is because more choices for actions result in less predictable decisions for each person in the network, making suggestions more valuable from an influence perspective.

We next study the recommender's influence in terms of *how responsive* the users in the social network are. We posit that responsive users will often react by taking actions, i.e, will avoid the output e. Hence, the degree of indifference in our model is dominated by the channel parameter ρ. We fix the parameters $\beta = 0.3$, $\alpha = 0.6$, $\theta = 0.3$, $\mu = 0.2$, $\phi = 0.2$, and $\rho = 0.2$, which correspond to moderate trust levels in Fig. 4, and control the degree of *indifference* of the users via ρ. For consistency, in doing so, we keep γ proportional to the changes in ρ by setting $\gamma = (1 - \rho)/2$. We present the maximum influence of the recommender with respect to network indifference, indicated by ρ, in Fig. 5. We observe that a recommender has greater influence in a responsive network, i.e., when ρ is small. However, in comparison to Fig. 4, the effect of indifference is not as dramatic as the effect of the recommender's trustworthiness.

V. Conclusion and Discussion

We proposed a recommendation model with underlying social interactions, where individuals are influenced by their previous decisions or the decisions of previous users. We characterized the fundamental limits of the recommender's influence on the network, which is measured by the information contained in the suggestions. Utilizing the equivalence of this model to a finite state channel with feedback, we determined the optimal recommendation pattern, which is a stationary strategy based on the previous decision. We observed how the influence is impacted by the recommender's reputation. Future directions include considering larger graphs with complex relationship structures and channel models that capture different aspects of human behavior and social interaction.

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