The Semantic Communication Game

Başak Güler, *Member, IEEE*, Aylin Yener, *Fellow, IEEE*, and Ananthram Swami, *Fellow, IEEE*

***Abstract*—This paper proposes a communication framework where meanings of transmitted codewords over a noisy channel are explicitly taken into account. Furthermore, such communication takes place in the presence of an external entity, i.e., an agent, that can influence the receiver. The agent may be adversarial or helpful, and its true nature is unknown to the communicating parties. Actions taken by the agent are governed by its nature to aim to improve/deteriorate the communication performance. We characterize the optimal transmission policies to minimize the end-to-end average semantic error, that we define as the expected error between meanings of intended and recovered messages, under the uncertainty of agent's true intentions. To do so, we first formulate the communication problem as a Bayesian game, and investigate the conditions under which a Bayesian Nash equilibrium exists. Next, we consider a dynamic communication scenario in which parties take actions sequentially, forming beliefs about the other party. By formulating this setting as a sequential game, we investigate the structure of the belief system and strategy profiles at equilibrium. Our results indicate that word semantics are instrumental in assessing communication performance when messages carry meanings, and optimal communication strategies are strongly influenced by the communicating parties' beliefs.**

***Index Terms*—Semantic communication, transmit codeword assignment, side information, game theory, Bayesian games, social influence.**

I. Introduction

PERFORMANCE criteria in conventional communication systems are based on error rates that are agnostic to the semantics of communicated messages. In fact, communication protocols that operate in the physical layer do not take into account the difference between the meanings of transmitted and recovered messages at all, but rather are concerned with the engineering problem of reliably communicating sequences of bits [3]. Emergent networks, e.g., the Internet of Things (IoT) paradigm, enable advanced connectivity between humans and machines [4], [5], where interaction occurs between parties of diverse backgrounds, interests, and inference capabilities. For such systems, reliable communication implies that the intended meaning of the messages is preserved at reception. In effect, these networks allow interaction at a level so that the communicating parties can form social relationships [6]. These factors motivate a new approach that molds physical and application layer metrics into one, i.e., a novel performance criterion that takes into account the meanings of the communicated messages, as well as utilizing this measure in a setting that abstracts the impact of social influence.

This paper introduces an approach in which the semantic content of the messages to be communicated over a noisy channel is taken into account in optimizing the performance of a communication system. Consider the following motivating example. In a conventional communication system, errors between semantically similar words, such as *car* and *automobile*, are treated equally as semantically distant words, such as *car* and *computer*. On the other hand, the meaning of *car* is much closer to *automobile* than *computer*. In reality, if one wishes to transmit the binary sequence corresponding to the message *car*, and if the channel errors prevent its recovery, it would be much better if the erroneous received sequence were decoded as *automobile* as compared to *computer*. We study mechanisms to achieve this, i.e., how to reliably communicate the meanings of messages through a noisy channel. To do so, we propose a novel performance metric that measures how accurately the meanings of messages are recovered. We view the semantic error caused by the noisy channel as the distance between the meanings of transmitted and received messages. This is in contrast with declaring an error when a message other than the original is recovered regardless of its meaning. Our semantic error measure is based on the notion of semantic similarity, which quantifies the distance between the meanings of two words [7]–[10], for instance, synonyms incur very small if any semantic error. Semantic similarity measures are widely used in natural language processing, artificial intelligence, computational linguistics, and information retrieval. By minimizing a semantic error measure based on the dissimilarity of word meanings, we determine the optimal transmission policies to best preserve the meanings of recovered messages.

In addition to establishing the semantic error metric for communication performance, we model the impact of social influence on how the messages are interpreted by considering an external influential entity, i.e., an agent, in our communication network. This agent can influence the receiver, in particular, how it decodes the received signals, by providing side, e.g., context, information. The exact nature of the agent, whether

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B. Güler was with the Department of Electrical Engineering, Pennsylvania State University, University Park, PA 16802 USA. She is now with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089 USA (e-mail: bguler@usc.edu).

A. Yener is with the Department of Electrical Engineering, Pennsylvania State University, University Park, PA 16802 USA (e-mail: yener@ee.psu.edu).

A. Swami is with the U.S. Army Research Laboratory, Adelphi, MD 20783 USA (e-mail: a.swami@ieee.org).

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adversarial or helpful, is unknown to the communicating parties, and is treated as a random variable. An adversarial agent, for instance, would target causing errors in message recovery, while a helpful one would act to facilitate effective communication. Hence, it is beneficial to tailor the transmission policies to take into account the possible intentions of the agent.

We model the interaction between the communicating parties and the agent by utilizing game theory, in particular, as a Bayesian game [11], [12]. Bayesian games consider static games with incomplete information due to uncertainty about the characteristics of one or more players. The objective of each player, i.e., its payoff function, is determined by its characteristics. Bayesian games can be extended to model dynamic scenarios, in which players take actions sequentially in multiple rounds. Games of this type are called sequential games with incomplete information. The sequential game has perfect information if the actions taken by each player are observed by all other players, otherwise it has imperfect information [12].

Bayesian games have been useful in modeling a diverse range of problems in communication networks in which interacting parties have incomplete information regarding various network parameters [13]–[22]. In [13], Bayesian games have been leveraged in network intrusion detection, for modeling the interaction between defending and attacking parties. References [14] and [15] utilize Bayesian games in the context of dynamic spectrum access between primary and secondary users, to identify the optimal spectrum monitoring strategies for detecting potential thievery of spectral resources by the secondary user. Wireless spectrum utilization is investigated in [16] in which a Bayesian game is designed to model the interaction between selfish communicating parties that operate in the same frequency band, in which the parties have incomplete information about the channel conditions of one another. The time-invariant channel setup considered in [16] is extended in [17] to take into account channel variations as well as channel estimation errors. In [18], a Bayesian game is formulated to study resource allocation in a fading multiple access channel, where the communicating parties aim to maximize their average achievable rates with incomplete information regarding the fading channel gains. Reference [19] applies the Bayesian game model to the downlink power allocation scenario in a two-tier heterogeneous cellular network, in which the macrocell base station has incomplete information about the channel between the femtocell base station and the femtocell user. Bayesian games have also found applications in signal processing for communications [21] and in adaptive multi-agent sensing scenarios [22]. In a different line of work, Bayesian games have been incorporated to design robust distributed wireless communication protocols in the presence of jammers, in which the interacting parties have incomplete information about the network conditions, such as the identities of other users, traffic dynamics, or characteristics of the physical channel, e.g., channel gain or noise [23]–[25], or whether the jammer is physically present or absent [26]. Reference [27] utilizes a Bayesian game to model an underwater acoustic sensor network in which nodes communicate in the presence of an adversary, and incomplete information results from the uncertainty about the nodes' exact position. A cognitive radio network is considered in [28] in which the power allocation strategies are studied for the secondary user to meet a minimum signal-to-interference-plus-noise ratio (SINR), in the presence of a jammer. In this model, the interaction between the secondary user and the jammer is modeled as a Bayesian game with power budget constraints and incomplete information regarding the channel gains.

We view the semantic communication problem with social influence as a game with incomplete information, played between the encoder/decoder and the agent. The encoder/decoder pair wishes to minimize the average semantic error between the meanings of transmitted and recovered messages. The agent depending on its nature may either act to aid or work against this goal. For the static (Bayesian) scenario in which both sides take their actions simultaneously, we investigate the conditions under which a pure strategy Bayesian Nash equilibrium exists, and characterize the mixed strategy Bayesian Nash equilibrium. After showing that finding the encoding and decoding strategies to minimize the average semantic error is NP-hard, we propose two algorithms based on simulated annealing and alternating optimization.

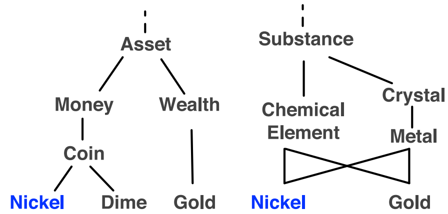
We next extend the communication scenario into a dynamic one, in which the agent and the communicating parties can take actions sequentially. We demonstrate a sequential equilibrium under which the agent *signals* its true nature to the communicating parties, by always choosing separate actions for when it is adversarial or helpful. In our numerical studies, we determine the equilibrium strategies as well as the structure of the encoding and decoding functions that minimize the average semantic error.

Our results confirm that judicious transmission policies can significantly reduce the semantic errors that occur between the meanings of intended and recovered words. In addition, optimal strategies are greatly influenced by the belief of the interacting parties about the true nature of the influential entity. To this end, transmission policies depend on the bias of the interacting parties, i.e., whether the individual is believed to be adversarial or helpful.

The remainder of the paper is organized as follows. In Section II, we overview the notion of semantic similarity. Section III introduces our system model. The Bayesian game formulation is described in Section IV. Section V analyzes the Nash equilibrium. Section VI studies the structure of the optimal encoding and decoding strategies. The dynamic communication game model is introduced in Section VII. Numerical results are provided in Section VIII. The paper is concluded in Section IX.

II. Semantic Similarity: A Brief Overview

Words can take different meanings in different contexts. A *concept* refers to the specific meaning that a word takes in a certain context. For instance, the word *nickel* may refer to a five-cent coin or a chemical substance. Semantic relations between different concepts are often represented by using word taxonomies [29], such as the WordNet taxonomy illustrated in Fig. 1a. Semantic similarity quantifies how similar two *concepts* are [7]–[10]. Word similarity quantifies the semantic



(a)

$sim(\cdot,\cdot)$	car	automobile	coast	shore	bird
car	1	1	0.1	0.111	0.125
automobile	1	1	0.25	0.2	0.167
coast	0.1	0.25	1	0.5	0.143
shore	0.111	0.2	0.5	1	0.167
bird	0.125	0.167	0.143	0.167	1
crane	0.143	0.125	0.125	0.143	0.25

(b)

Fig. 1. (a) A word taxonomy fragment from [7]. Concepts are related to each other using an *is-a* relationship, for instance, dime *is a* coin. The same word may appear at several places in the taxonomy as instances of different concepts, such as *nickel* appearing under *coin* (thus taking the meaning of a five-cent coin) versus *chemical substance*. (b) Examples of edge-based similarity values between various words.

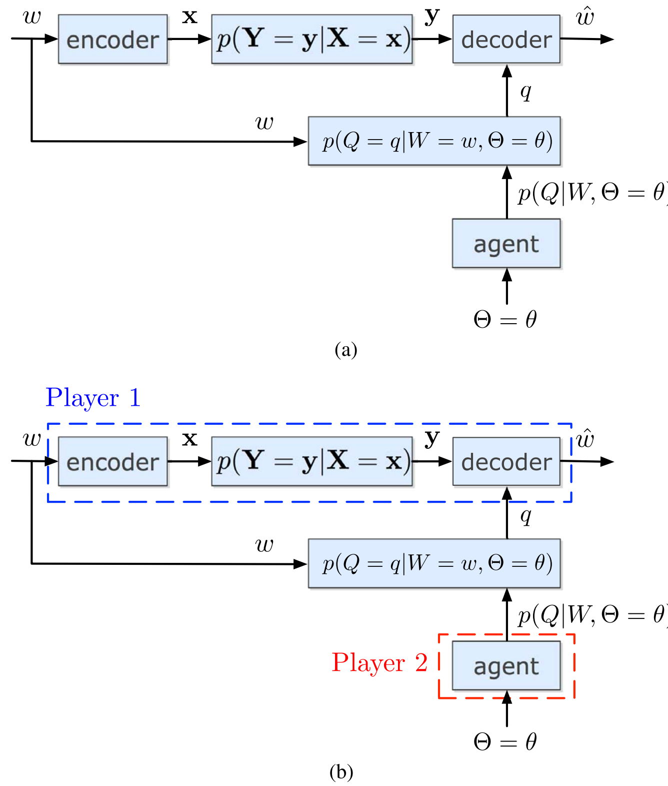


Fig. 2. (a) Semantic communication channel. (b) Players of the Bayesian game formulation.

similarity between a pair of *words*, with synonyms having the highest value. Word similarity measures are often based on a word taxonomy, or statistics from a large set of texts, i.e., a text corpus [7]–[10]. The main semantic similarity metrics are node-based similarity and edge-based similarity.

Node-based similarity between two concepts is measured by the information content of their lowest common subsumer [7]. A lowest common subsumer refers to the concept that has the shortest distance from the two. For example, coin and money are both subsumers of nickel and dime, but coin is lower subsumer than money. The information content of a concept is inversely proportional to its empirical frequency in the corpus. That is, concepts that appear less frequently in text have a higher information content. Edge-based similarity utilizes the geometric distance between two nodes in the word taxonomy to find the semantic similarity between two concepts [9]. This measure is inversely proportional to the length of the shortest path between two concepts. Similarity between two concepts increases as the path connecting the two nodes becomes shorter. Similarity between two words is defined as the maximum of all similarity values between the concepts corresponding to them. Fig. 1b presents examples of edge-based similarity values between various words evaluated using the NLTK language processing package [30].

III. System Model

Notation: In the following, uppercase letters represent random variables, whereas lowercase letters correspond to their realizations. We use $\mathbf{x} = (x_1, \ldots, x_n)$ for a vector of length $n \in \mathbb{Z}^+$. $\mathcal{X}$ represents a set with cardinality $|\mathcal{X}|$ and $\mathcal{X}^n$ is the n-fold Cartesian product of $\mathcal{X}$. $\mathbb{E}[\,\cdot\,]$ denotes the expectation operation.

We consider the communication scenario in Fig. 2a. In this scenario, the encoder observes a message (word) w from a finite set $\mathcal{W}$ with probability $p(W = w)$. It then maps w to a channel input $\mathbf{x} = (x_1, \ldots, x_n)$ using an encoding function $g : \mathcal{W} \to \mathcal{X}^{(n)}$, where $\mathcal{X}^{(n)} \subseteq \mathcal{X}^n$ and $\mathcal{X}$ is a finite alphabet. The set of all such encoding functions is denoted by $\mathcal{G}$. A noisy channel exists between the encoder and the decoder, which is characterized by the conditional probability $p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x})$. The channel output $\mathbf{y} = (y_1, \ldots, y_n)$ is a vector of length n from the set $\mathcal{Y}^{(n)}$ such that $\mathcal{Y}^{(n)} \subseteq \mathcal{Y}^n$ where $\mathcal{Y}$ is a finite alphabet.

The agent's true nature, which could be either adversarial or helpful, is characterized by a random variable Θ governed by the probability,

$$p(\Theta = \theta) = \begin{cases} \alpha & \text{if} \quad \theta = a \\ 1-\alpha & \text{if} \quad \theta = h \end{cases} \tag{1}$$

for some $0 < \alpha < 1$, where $\theta = a$ if the agent is adversarial and $\theta = h$ if the agent is helpful. We note that α can take any value between (0,1), hence comes from an infinite set. However, its value is fixed and is known by both the agent and the encoder/decoder pair. Accordingly, the encoder and the decoder know the distribution in (1), but do not know the actual realization θ, i.e., the true nature of the agent. It is assumed that α is a parameter that can be estimated via various information sources, such as by monitoring the data on the past behavior of the agent, or gathering information from different applications on its activity patterns. The agent can take actions to influence how the decoder perceives the received information. The chosen action depends on the agent's true nature, i.e., adversarial or helpful. In particular, the agent draws a random variable $p(Q|W, \Theta = \theta)$ from a finite set $\mathcal{P}$. Each element of $\mathcal{P}$ identifies a conditional probability distribution over the set of contexts $\mathcal{Q}$ for each $w \in \mathcal{W}$, so that

$$\sum_{q \in \mathcal{Q}} p(Q = q| W = w, \Theta = \theta) = 1, \tag{2}$$

where $p(Q = q| W = w, \Theta = \theta) \geq 0$ for all $q \in \mathcal{Q}$. The decoder then observes a context q governed by the probability $p(Q = q| W = w, \Theta = \theta)$. That is, the context information observed by the decoder is affected by the distribution chosen by the agent. In that sense, the agent can influence the decoder by controlling the distribution $p(Q|W, \Theta = \theta)$. As an example, consider a communication scenario in which the encoder and decoder are smart devices deployed with sensors to monitor their environment. Suppose that the agent can place the decoder in one of the following two environments, an underwater environment vs. a desert environment. Depending on the environment it is placed in, the decoder will take different measurements and will have a different view of the context in which the communication is taking place. For instance, the animal and plant habitat in an underwater environment is different than that of a desert environment. This may in turn lead to different interpretations of the received message.

Lastly, we assume that the words observed by the encoder are independent from the true nature of the agent, i.e.,

$$p(W = w|\Theta = \theta) = p(W = w). \tag{3}$$

The decoder recovers a word $\hat{w} \in \mathcal{W}$ from the channel output $\mathbf{y}$ and context q using a decoding function $h : \mathcal{Y}^{(n)} \times \mathcal{Q} \rightarrow \mathcal{W}$. The set of all valid decoding functions is denoted as $\mathcal{H}$. The channel output, when conditioned on the channel input, is independent from the rest of the random variables in the model, i.e., we have that $Y - X - WQ\Theta$ form a Markov chain,

$$\begin{aligned} p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x}, W = w, Q = q, \Theta = \theta) \\ = p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x}) \end{aligned} \tag{4}$$

where $\mathbf{x} = g(w)$. The sets $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{P}$ are finite.

We define the semantic distance between two words as,

$$d(w, \hat{w}) = 1 - sim(w, \hat{w}), \quad w, \hat{w} \in \mathcal{W}, \tag{5}$$

where $0 \leq sim(w, \hat{w}) \leq 1$ denotes the semantic similarity between w and $\hat{w}$. Given $\Theta = \theta$, we define the average semantic error corresponding to $(g, h) \in \mathcal{G} \times \mathcal{H}$ and $p(Q|W, \Theta = \theta) \in \mathcal{P}$ as,

$$\begin{aligned} &D_\theta((g, h), p(Q|W, \Theta = \theta)) \\ &= \sum_{w \in \mathcal{W}, q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}} p(W = w, Q = q, \mathbf{Y} = \mathbf{y}|\Theta = \theta) d(w, h(\mathbf{y}, q)) \quad (6) \\ &= \sum_{w \in \mathcal{W}, q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}} \sum_{\mathbf{x} \in \mathcal{X}^{(n)}} p(W = w, Q = q, \mathbf{Y} = \mathbf{y}, \mathbf{X} = \mathbf{x}|\Theta = \theta) \\ &\qquad \times d(w, h(\mathbf{y}, q)) \quad (7) \\ &= \sum_{w \in \mathcal{W}, q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}} \sum_{\mathbf{x} \in \mathcal{X}^{(n)}} p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x}, W = w, Q = q, \Theta = \theta) \\ &\qquad \times p(\mathbf{X} = \mathbf{x}| W = w, Q = q, \Theta = \theta) p(Q = q, W = w|\Theta = \theta) \\ &\qquad \times d(w, h(\mathbf{y}, q)) \quad (8) \\ &= \sum_{\substack{w \in \mathcal{W}, q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}: \\ \mathbf{x} = g(w)}} p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x}, W = w, Q = q, \Theta = \theta) \\ &\qquad \times p(Q = q| W = w, \Theta = \theta) p(W = w|\Theta = \theta) d(w, h(\mathbf{y}, q)) \quad (9) \\ &= \sum_{\substack{w \in \mathcal{W}, q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}: \\ \mathbf{x} = g(w)}} p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x}) p(Q = q| W = w, \Theta = \theta) \\ &\qquad \times p(W = w) d(w, h(\mathbf{y}, q)) \quad (10) \end{aligned}$$

where (8) follows from the chain rule of probability, (9) holds since $p(\mathbf{X} = \mathbf{x}| W = w, Q = q, \Theta = \theta) = 1$ if and only if $\mathbf{x} = g(w)$ for the deterministic encoding function g, and (10) follows from (3) and (4).

IV. Bayesian Game Formulation

In this section, we describe our game-theoretic formulation for the communication model introduced in Section III. Consider a two-player static Bayesian game. Player 1 is the system designer who selects the encoding and decoding protocols. Player 2 represents the agent. An illustration of the two players is depicted in Fig. 2b. One can think of players 1 and 2 as an abstraction of the following communication scenario. Player 1 wishes to design a system to transmit information about the traffic status in one location to a user. The user has access to a sensor, e.g., camera, whose measurements can be utilized while interpreting the received information. Player 2 controls the location and the settings of the camera, which allows her to affect how the measurements are interpreted by the user. As such, the communication performance depends on the characteristic of player 2, i.e., whether it is adversarial or helpful.

The uncertainty about a player's true nature is termed in the game theory literature as its *type* [31]. We adopt the same terminology in the sequel. Player 1 has a single (fixed) type that is known by both parties. Player 2 belongs to one of two possible types, *adversary*, denoted by $\Theta = a$, or *helpful*, denoted by $\Theta = h$. The uncertainty about the type of player 2 is characterized by the probability $p(\Theta = \theta)$ from (1). The type of player 2 is known only by itself.

The strategy set of player 1 consists of all pairs of encoding and decoding functions $(g, h) \in \mathcal{G} \times \mathcal{H}$, and is denoted by $\mathcal{S}_1 \triangleq \mathcal{G} \times \mathcal{H}$. We represent the strategy chosen by player 1 by $s_1 \in \mathcal{S}_1$. The strategy set of player 2 consists of all the elements in $\mathcal{P}$ and is denoted as $\mathcal{S}_2 \triangleq \mathcal{P}$. We represent the strategy chosen by player 2 by $s_2(\Theta) \in \mathcal{S}_2$ given its type $\Theta \in \{a, h\}$. That is, player 2 follows the strategy $s_2(a)$ if it is adversarial and $s_2(h)$ if it is helpful.

We next introduce the payoff functions of the two players. Player 1 wants to choose the encoding and decoding strategy that minimize the average semantic error from (10). Accordingly, we define the payoff function of player 1 as,

$$u_1(s_1, s_2(\Theta), \Theta) = \begin{cases} -D_a(s_1, s_2(a)) & \text{if } \Theta = a \\ -D_h(s_1, s_2(h)) & \text{if } \Theta = h \end{cases} \tag{11}$$

where $s_1 \in \mathcal{S}_1$, $s_2(\Theta) \in \mathcal{S}_2$ for $\Theta \in \{a, h\}$, and $D_\theta(s_1, s_2(\theta))$ is as defined in (10). Player 1 wishes to maximize its expected payoff

$$\begin{aligned}\mathbb{E}[u_1(s_1, s_2(\Theta), \Theta)] \\ &= -\sum_{\theta\in\{a,h\}} p(\Theta = \theta) D_\theta(s_1, s_2(\theta)), \quad (12)\\ &= -\alpha D_a(s_1, s_2(a)) - (1-\alpha) D_h(s_1, s_2(h)) \quad (13)\end{aligned}$$

where the expectation is taken over the distribution of the types of the agent.

The payoff function of player 2 depends on its type. Specifically, if player 2 is adversarial (helpful), it wishes to maximize (minimize) the average semantic error from (10). The payoff function for player 2 is then defined as,

$$u_2(s_1, s_2(\Theta), \Theta) = \begin{cases} D_a(s_1, s_2(a)) & \text{if } \Theta = a \\ -D_h(s_1, s_2(h)) & \text{if } \Theta = h \end{cases} \quad (14)$$

The two players choose their strategies independently before communication starts. We consider a non-cooperative game in which each player wishes to maximize its individual payoff. Both players are rational.

V. Bayesian Nash Equilibrium

A. Pure Strategy Nash Equilibrium

This section identifies sufficient conditions for the existence of a pure strategy Bayesian Nash equilibrium. Specifically, we let $(s_1^*, s_2^*(a), s_2^*(h))$ denote a pure strategy Bayesian Nash equilibrium, where the equilibrium conditions are given by,

$$\begin{aligned}&-\alpha D_a(s_1^*, s_2^*(a)) - (1-\alpha) D_h(s_1^*, s_2^*(h)) \\ &\geq -\alpha D_a(s_1, s_2^*(a)) - (1-\alpha) D_h(s_1, s_2^*(h)), \ \ \forall s_1 \in \mathcal{S}_1 \quad (15)\end{aligned}$$

$$D_a(s_1^*, s_2^*(a)) \geq D_a(s_1^*, s_2(a)), \ \ \forall s_2(a) \in \mathcal{S}_2, \quad (16)$$

$$-\ D_h(s_1^*, s_2^*(h)) \geq -D_h(s_1^*, s_2(h)), \ \ \forall s_2(h) \in \mathcal{S}_2 \quad (17)$$

where $s_1^* \in \mathcal{S}_1$ is player 1's equilibrium strategy, $s_2^*(a) \in \mathcal{S}_2$ is player 2's equilibrium strategy if she is adversarial, and $s_2^*(h) \in \mathcal{S}_2$ is player 2's equilibrium strategy if she is helpful. Conditions (15)-(17) ensure that both players play their best responses against each other at equilibrium. Specifically, (15) ensures that player 1 selects the encoding/decoding function that maximizes (13), whereas (16) and (17) ensure that the agent chooses the strategy that maximizes (14) when it is adversarial or helpful, respectively.

In the following, we identify two pure strategy Bayesian Nash equilibriums that occur when $\alpha \leq \underline{\alpha}$ and $\alpha \geq \bar{\alpha}$, respectively, for some thresholds $\underline{\alpha} \leq \bar{\alpha}$. Theorem 1 shows that if the likelihood of the agent being a helpful one is sufficiently high ($\alpha \leq \underline{\alpha}$), the equilibrium policies are the same as the setup in which the agent is indeed helpful. Theorem 2 shows that if the likelihood of the agent being an adversarial one is sufficiently high ($\alpha \geq \bar{\alpha}$), the equilibrium policies are the same as the setup in which the agent is indeed adversarial.

We first demonstrate a pure strategy Bayesian Nash equilibrium for $\alpha \leq \underline{\alpha}$ and show that the encoding/decoding strategy at equilibrium is the same as one that would be selected if the agent was *known* to be helpful.

Theorem 1: Define $(s_1^*, s_2^*(a), s_2^*(h))$ such that

$$s_1^* = \arg\min_{s_1\in\mathcal{S}_1} D_h(s_1, s_2^*(h)), \quad (18)$$

$$s_2^*(a) = \arg\max_{s_2(a)\in\mathcal{S}_2} D_a(s_1^*, s_2(a)), \quad (19)$$

$$s_2^*(h) = \arg\min_{s_2(h)\in\mathcal{S}_2} D_h(s_1^*, s_2(h)). \quad (20)$$

Then, $(s_1^*, s_2^*(a), s_2^*(h))$ denotes a pure strategy Bayesian Nash equilibrium whenever the following conditions are satisfied:

$$\text{i) } \alpha \leq \underline{\alpha} \quad \text{for some} \quad \underline{\alpha} \leq \min_{\substack{s_1\in\mathcal{S}_1 : s_1 \neq s_1^* \\ \beta(s_1)>0, \kappa(s_1)>0}} \frac{\beta(s_1)}{\kappa(s_1)}, \quad (21)$$

$$\text{ii) If } \beta(s_1) = 0 \text{ for some } s_1 \in \mathcal{S}_1 \text{ with } s_1 \neq s_1^*, \text{ then } \kappa(s_1) \leq 0, \quad (22)$$

where $\beta(s_1) = D_h(s_1, s_2^*(h)) - D_h(s_1^*, s_2^*(h))$ and $\kappa(s_1) = \beta(s_1) + D_a(s_1^*, s_2^*(a)) - D_a(s_1, s_2^*(a))$.

Proof: We prove this result by showing that whenever (21) and (22) hold, $(s_1^*, s_2^*(a), s_2^*(h))$ satisfies the equilibrium conditions (15)-(17). Using (13), we first find player 1's expected payoff if it follows the strategy $s_1 \in \mathcal{S}_1$ while player 2 plays $s_2^*(a)$ if adversarial and $s_2^*(h)$ if helpful,

$$-\ \alpha D_a(s_1, s_2^*(a)) - (1-\alpha) D_h(s_1, s_2^*(h)). \quad (23)$$

To prove that s_1^* is player 1's best response, it remains to show for all $s_1 \in \mathcal{S}_1$ that,

$$\begin{aligned}&-\alpha D_a(s_1^*, s_2^*(a)) - (1-\alpha) D_h(s_1^*, s_2^*(h)) \\ &\quad - (-\alpha D_a(s_1, s_2^*(a)) - (1-\alpha) D_h(s_1, s_2^*(h))) \\ &= \beta(s_1) - \alpha\kappa(s_1) \geq 0. \quad (24)\end{aligned}$$

For $s_1 = s_1^*$, (24) holds by definition. For $s_1 \neq s_1^*$, if $\beta(s_1) > 0$ and $\kappa(s_1) > 0$, we have $\beta(s_1) \geq \alpha\kappa(s_1)$ from (21), hence (24) holds. If instead $\beta(s_1) = 0$, we have from (22) that (24) again holds. From (18), we observe that

$$D_h(s_1^*, s_2^*(h)) \leq D_h(s_1, s_2^*(h)) \quad (25)$$

and therefore $\beta(s_1) \geq 0$ for all $s_1 \in \mathcal{S}_1$. Hence, the only remaining case we need to investigate is when $\beta(s_1) > 0$, $\kappa(s_1) \leq 0$, which also satisfies (24) since $\alpha > 0$.

Next, we find from (14) that if player 2 is adversarial,

$$D_a(s_1^*, s_2^*(a)) = \max_{s_2(a)\in\mathcal{P}} D_a(s_1^*, s_2(a)) \geq D_a(s_1^*, s_2(a)) \quad (26)$$

for all $s_2(a) \in \mathcal{P}$, hence (16) is satisfied. If instead player 2 is helpful, we find from (14) that,

$$\begin{aligned}-\ D_h(s_1^*, s_2^*(h)) &= -\min_{s_2(h)\in\mathcal{P}} D_h(s_1^*, s_2(h)) \\ &\geq -D_h(s_1^*, s_2(h)) \quad (27)\end{aligned}$$

for all $s_2(h) \in \mathcal{P}$, hence (17) is also satisfied.

We have now shown that s_1^* is player 1's best response against player 2's strategies $s_2^*(a)$ and $s_2^*(h)$, where player 2 plays $s_2^*(a)$ if adversarial and $s_2^*(h)$ if helpful. At the same time, $s_2^*(a)$ and $s_2^*(h)$ are player 2's best response against player 1's strategy s_1^*, for when player 2 is adversarial or helpful, respectively. As a result, $(s_1^*, s_2^*(a), s_2^*(h))$

from (18)–(20) is a pure strategy Bayesian Nash equilibrium. ∎

Next, we identify a pure strategy Bayesian Nash equilibrium when $\alpha \geq \bar{\alpha}$ and show that the encoding/decoding strategy at equilibrium is the same as one that would be selected if the agent was *known* to be adversarial.

Theorem 2: Define $(s_1^*, s_2^*(a), s_2^*(h))$ such that

$$s_1^* = \arg\min_{s_1 \in \mathcal{S}_1} D_a(s_1, s_2^*(a)), \tag{28}$$

$$s_2^*(a) = \arg\max_{s_2(a) \in \mathcal{S}_2} D_a(s_1^*, s_2(a)), \tag{29}$$

$$s_2^*(h) = \arg\min_{s_2(h) \in \mathcal{S}_2} D_h(s_1^*, s_2(h)). \tag{30}$$

Then, $(s_1^*, s_2^*(a), s_2^*(h))$ denotes a pure strategy Bayesian Nash equilibrium if

$$\alpha \geq \bar{\alpha} \quad \text{for some} \quad \bar{\alpha} \geq \max_{\substack{s_1 \in \mathcal{S}_1 : s_1 \neq s_1^* \\ \beta(s_1) < 0, \kappa(s_1) < 0}} \frac{\beta(s_1)}{\kappa(s_1)} \tag{31}$$

where $\beta(s_1) = D_h(s_1, s_2^*(h)) - D_h(s_1^*, s_2^*(h))$ and $\kappa(s_1) = \beta(s_1) + D_a(s_1^*, s_2^*(a)) - D_a(s_1, s_2^*(a))$.

Proof: The proof follows the same lines as (23)-(27). We initially show that s_1^* is player 1's best response by demonstrating that it satisfies (24). For any $s_1 \neq s_1^*$, if $\beta(s_1) < 0, \kappa(s_1) < 0$, we have $\alpha\kappa(s_1) \leq \beta(s_1)$ from (31), therefore (24) holds. Next, we note that

$$\kappa(s_1) = \beta(s_1) - (D_a(s_1, s_2^*(a)) - D(s_1^*, s_2^*(a))) \leq \beta(s_1) \tag{32}$$

since $D_a(s_1, s_2^*(a)) \geq D_a(s_1^*, s_2^*(a))$ for all $s_1 \in \mathcal{S}_1$ from (28). As a result, we only need to inspect the following remaining cases: i) $\beta(s_1) > 0$, $\kappa(s_1) > 0$, ii) $\beta(s_1) > 0$, $\kappa(s_1) = 0$, iii) $\beta(s_1) = 0$, $\kappa(s_1) < 0$, iv) $\beta(s_1) = 0$, $\kappa(s_1) = 0$, v) $\beta(s_1) > 0$, $\kappa(s_1) < 0$. We first observe that (24) holds for i) due to (32) and

$$\alpha\kappa(s_1) \leq \kappa(s_1) \leq \beta(s_1) \text{ for } 0 < \alpha < 1. \tag{33}$$

We next observe that (24) also holds for ii), iii), iv) and v) as $\alpha > 0$. As a result, $(s_1^*, s_2^*(a), s_2^*(h))$ is a pure strategy Bayesian Nash equilibrium. ∎

Remark 1: It is useful to note that the results in Theorems 1 and 2 follow a convex combination argument, as the agent is either a minimizer or a maximizer, the monotone behavior of the equilibrium policies are preserved while the prior on the agent's type changes, until certain thresholds are reached.

B. Mixed Strategy Nash Equilibrium

For the semantic communication game, a mixed strategy Bayesian Nash equilibrium always exists, as both players have a finite set of types and strategies [31], [32]. In this section, we characterize the structure of mixed strategies at equilibrium. We first define $\phi \in \Delta(\mathcal{S}_1)$ as a mixed strategy for player 1, where $\Delta(\mathcal{S}_1)$ is the set of all probability distributions over $\mathcal{S}_1$. Here ϕ assigns a probability to each $s_1 \in \mathcal{S}_1$ and we denote this probability by $\phi(s_1)$. Accordingly, $\phi(s_1) \geq 0$ for all $s_1 \in \mathcal{S}_1$ and $\sum_{s_1 \in \mathcal{S}_1} \phi(s_1) = 1$. Given $\Theta = \theta$, we next define $\phi_\theta \in \Delta(\mathcal{S}_2)$ as a mixed strategy for player 2 whose type is $\theta \in \{a, h\}$. Again, $\Delta(\mathcal{S}_2)$ is the set of all probability distributions over $\mathcal{S}_2$. That is, ϕ_θ assigns a probability to each element $s_2(\theta) \in \mathcal{S}_2$ and we denote this probability by $\phi_\theta(s_2(\theta))$. Accordingly, $\phi_\theta(s_2(\theta)) \geq 0$ for all $s_2(\theta) \in \mathcal{S}_2$, and $\sum_{s_2(\theta) \in \mathcal{S}_2} \phi_\theta(s_2(\theta)) = 1$.

At equilibrium, each player again wishes to maximize its expected payoff. For player 2, this expectation is over the distributions indicated by the mixed strategies. For player 1, the expectation is now over both the possible types of player 2 and the mixed strategies. Accordingly, $(\phi^*, \phi_a^*, \phi_h^*)$ is a mixed strategy Bayesian Nash equilibrium if

$$\begin{aligned}\phi^* = \arg\min_{\phi_1 \in \Delta(\mathcal{S}_1)} \Big(\alpha \sum_{s_1 \in \mathcal{S}_1} \sum_{s_2(a) \in \mathcal{S}_2} \phi(s_1)\phi_a^*(s_2(a)) \\ \times D_a(s_1, s_2(a), a)) + (1-\alpha) \sum_{s_1 \in \mathcal{S}_1} \sum_{s_2(h) \in \mathcal{S}_2} \phi(s_1)\phi_h^*(s_2(h)) \\ \times D_h(s_1, s_2(h), h)\Big),\end{aligned} \tag{34}$$

$$\phi_a^* = \arg\max_{\phi_a \in \Delta(\mathcal{S}_2)} \sum_{s_1 \in \mathcal{S}_1} \sum_{s_2(a) \in \mathcal{S}_2} \phi^*(s_1)\phi_a(s_2(a)) \times D_a(s_1, s_2(a), a), \tag{35}$$

$$\phi_h^* = \arg\min_{\phi_h \in \Delta(\mathcal{S}_2)} \sum_{s_1 \in \mathcal{S}_1} \sum_{s_2(h) \in \mathcal{S}_2} \phi^*(s_1)\phi_h(s_2(h)) \times D_h(s_1, s_2(h), h). \tag{36}$$

The conventional route to compute (34)-(36) is for each player to find the strategy that makes the other player indifferent to each one of their pure strategies. It is useful to note that the computational complexity of this problem requires careful consideration, and various algorithms have been proposed to address this challenge [33], [34]. The interpretations as to why players would adopt mixed strategies is an ongoing debate in game theory, common views include treating mixed strategies at equilibrium as the steady state of a game that is played repeatedly, or interpreting mixed strategies as a result of small variations in the payoffs of the players. We refer the reader to [31, Sec. 3.2] for a detailed discussion on this topic.

VI. Encoding and Decoding Strategies for Minimum Semantic Error

In this section, we fix the strategy of player 2 by letting $|\mathcal{P}| = 1$, and focus on the encoding/decoding functions that minimize the average semantic error from (10) for a given distribution between the words and contexts. We utilize in this section the following notation

$$p(Q|W) \triangleq p(Q|W, \Theta = a) = p(Q|W, \Theta = h) \tag{37}$$

to represent the unique element in $\mathcal{P}$. By using (37), we represent the average semantic error from (10) for this setting as,

$$\begin{aligned}&D((g,h), p(Q|W)) \\ &\triangleq D_a((g,h), p(Q|W, \Theta = a)) \quad (38)\\ &= D_h((g,h), p(Q|W, \Theta = h)) \quad (39)\end{aligned}$$

Algorithm 1 Alternating Minimization

Initialize g and h to arbitrary elements of $\mathcal{G}$ and $\mathcal{H}$, respectively.
$D_{old} \coloneqq \infty$; $D_{new} \coloneqq D((g,h), p(Q|W))$.
while $D_{old} - D_{new} > \epsilon$
  $D_{old} \coloneqq D_{new}$
  for all $q \in \mathcal{Q}$ and $\mathbf{y} \in \mathcal{Y}^{(n)}$ ▷ Optimal decoding function for fixed encoding function.
    Initialize $\hat{w}$ to an arbitrary element of $\mathcal{W}$, compute $f_d(q, \mathbf{y}, \hat{w})$ from (43).
    for all $w' \in \mathcal{W}$
      Compute $f_d(q, \mathbf{y}, w')$ from (43).
      if (43) is false **then** $\hat{w} \coloneqq w'$.
    Record $h(\mathbf{y}, q) = \hat{w}$.
  for all $w \in \mathcal{W}$ ▷ Optimal encoding function for fixed decoding function.
    Initialize $\mathbf{x}$ to an arbitrary element of $\mathcal{X}^{(n)}$, compute $f_e(w, \mathbf{x})$ from (45).
    for all $\mathbf{x}' \in \mathcal{X}^{(n)}$
      Compute $f_e(w, \mathbf{x}')$ from (45).
      if (45) is false **then** $\mathbf{x} \coloneqq \mathbf{x}'$.
    Record $g(w) = \mathbf{x}$.
  Calculate $D((g,h), p(Q|W))$ from (40) and set $D_{new} = D((g,h), p(Q|W))$.
return g and h

Algorithm 2 Simulated Annealing

Choose an initial state by assigning g and h to an arbitrary element of $\mathcal{G}$ and $\mathcal{H}$. Calculate $D((g,h), p(Q|W))$ from (40).
Initialize the melting temperature T_m and the freezing temperature T_f.
Initialize the maximum number of iterations N_{max}.
$T \coloneqq T_m$.
N:=1.
while $T > T_f$ **or** $N < N_{max}$
  N:=N+1
  $g_{temp} \coloneqq g$; $h_{temp} \coloneqq h$.
  Generate a Bernoulli random variable $K \sim$ *Bern*(1/2).
  if $K = 1$ **then** Pick $w \in \mathcal{W}$ uniformly at random, assign $g_{temp}(w)$ to a new random codeword from $\mathcal{X}^{(n)}$.
  else Choose $q \in \mathcal{Q}$ and $\mathbf{y} \in \mathcal{Y}^{(n)}$ uniformly at random, set $h_{temp}(\mathbf{y}, q)$ to a random word from $\mathcal{W}$. ▷ Perturb state.
  Calculate $D((g_{temp}, h_{temp}), p(Q|W))$ from (40).
  $\Delta D \coloneqq D((g_{temp}, h_{temp}), p(Q|W)) - D((g,h), p(Q|W))$
  if $\Delta D < 0$ **then** $g \coloneqq g_{temp}$; $h \coloneqq h_{temp}$
  else
    ($g \coloneqq g_{temp}$; $h \coloneqq h_{temp}$) with probability $e^{-\Delta D/T}$
    Reduce temperature.
return g and h

$$= \sum_{\substack{w \in \mathcal{W}, q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}: \\ \mathbf{x} = g(w)}} p(\mathbf{Y} = \mathbf{y} | \mathbf{X} = \mathbf{x}) \times p(Q = q | W = w) p(W = w) d(w, h(\mathbf{y}, q)). \tag{40}$$

The optimal encoding/decoding function pair (g^*, h^*) is given by

$$(g^*, h^*) = \min_{(g,h) \in \mathcal{G} \times \mathcal{H}} D((g,h), p(Q|W)). \tag{41}$$

To find the optimal decoding rule for a fixed encoding function g, one needs to find $h \in \mathcal{H}$ that minimizes (40),

$$D((g,h), p(Q|W)) = \sum_{q \in \mathcal{Q}} p(Q = q) \sum_{\mathbf{y} \in \mathcal{Y}^{(n)}} \sum_{\substack{w \in \mathcal{W}: \\ \mathbf{x} = g(w)}} p(W = w | Q = q) \times p(\mathbf{Y} = \mathbf{y} | \mathbf{X} = \mathbf{x}) d(w, h(\mathbf{y}, q)), \tag{42}$$

where $p(Q = q) = \sum_{w \in \mathcal{W}} p(Q = q | W = w) p(W = w)$ and $p(W = w | Q = q) = \frac{p(Q=q|W=w)p(W=w)}{p(Q=q)}$. From (42), the optimal decoding rule can be determined as follows. For each $q \in \mathcal{Q}$ and $\mathbf{y} \in \mathcal{Y}^{(n)}$, assign $h(\mathbf{y}, q) = \hat{w}$ if

$$f_d(q, \mathbf{y}, \hat{w}) \leq f_d(q, \mathbf{y}, w') = \sum_{w \in \mathcal{W}: \mathbf{x} = g(w)} p(W = w | Q = q) \times p(\mathbf{Y} = \mathbf{y} | \mathbf{X} = \mathbf{x}) d(w, w') \tag{43}$$

for all $w' \in \mathcal{W}$. Similarly, to find the optimal encoding rule for a fixed decoding function h, one needs to find $g \in \mathcal{G}$ that minimizes (40),

$$D((g,h), p(Q|W)) = \sum_{w \in \mathcal{W}} p(W = w) \sum_{\substack{q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}: \\ \mathbf{x} = g(w)}} p(Q = q | W = w) \times p(\mathbf{Y} = \mathbf{y} | \mathbf{X} = \mathbf{x}) d(w, h(\mathbf{y}, q)), \tag{44}$$

leading to the encoding rule $g(w) = \mathbf{x}$ if

$$f_e(w, \mathbf{x}) \leq f_e(w, \mathbf{x}') = \sum_{\substack{q \in \mathcal{Q}, \mathbf{y} \in \mathcal{Y}^{(n)}: \\ \mathbf{x}' = g(w)}} p(Q = q | W = w) p(\mathbf{Y} = \mathbf{y} | \mathbf{X} = \mathbf{x}') \times d(w, h(\mathbf{y}, q)) \tag{45}$$

for all $\mathbf{x}' \in \mathcal{X}^{(n)}$.

Finding the encoding/decoding functions that minimize the average semantic error is NP-hard, the details of which is provided in Appendix A. We propose two algorithms in the sequel to address the computational intensity. Algorithm 1 is based on alternating optimization with an error tolerance $\epsilon > 0$. Starting from a random initial assignment of the encoding and decoding functions, the algorithm iterates between two steps until convergence. In one step, the optimal decoding function is determined from (43) by fixing the encoding function. In the subsequent step, the optimal encoding function is determined via (45) by fixing the decoding function. Since the average semantic error is non-increasing at each iteration, and is bounded from below by zero, the algorithm converges, it may, however, converge to a local optimum. Algorithm 2 is a probabilistic metaheuristic based on simulated annealing [35], an effective approximate method to reduce the complexity in a large search space for combinatorial problems [36]–[38]. Its main idea is to, starting from an initial state, i.e., encoder/decoder assignment, *perturb* the state at each round by modifying the assignment. The new assignment is kept if it performs better than the old one, otherwise, is kept with a probability depending on a temperature parameter, which is reduced gradually, making the fluctuations less random as the algorithm progresses. The algorithm stops when the temperature reaches a minimum. The performance comparisons of the two algorithms are detailed in Section VIII.

VII. Dynamic Communication Game

We now consider the dynamic communication scenario where the encoder/decoder pair (player 1) and the agent (player 2) take actions sequentially. Specifically, at one round the agent takes an action, observed by the encoder/decoder pair. In the subsequent round, the encoder/decoder pair takes an action. The actions chosen by the encoder/decoder pair cannot be observed by the agent, hence the agent is oblivious to the selected encoding/decoding strategy. The true nature of the agent, adversarial or helpful, is again unknown to the encoder/decoder pair, hence the game has incomplete information [12]. Since the actions of some players cannot be observed others, the game also has imperfect information [12].

In the dynamic game, care must be taken to address the dimensionality of the strategy spaces of the players. The number of possible encoding strategies for player 1, for instance, is exponential in the number of words to be encoded. To this end, we consider two possible actions for player 2. The first action induces a distribution under which every context is equally likely given each word, i.e., the agent selects a random variable $p(Q|W, \Theta = \theta)$ such that $p(Q = q|W = w, \Theta = \theta) = \frac{1}{|\mathcal{Q}|}$ for every $w \in \mathcal{W}$ and $q \in \mathcal{Q}$. As such, this action prevents the decoder from using context information while decoding. The second action, on the other hand, induces a distribution that enables the decoder to achieve a better decoding performance by using context information. Formally, we let $\mathcal{S}_2 = \mathcal{P} = \{p_B, p_G\}$ denote the strategy set for player 2, where p_B and p_G corresponds to the former (bad) and latter (good) type of distribution.

For the actions of player 1, we consider two encoding strategies. The first one is a robust encoding strategy that minimizes the average semantic error against a bad distribution, i.e., p_B, by assigning each pair of words with different meanings to a different codeword. Hence, errors that may be caused at the decoder are solely due to channel noise, and the decoder does not need to rely on context information for decoding. The second one is an encoding strategy for a good distribution, i.e., p_G, which assigns the words that have a low chance of appearing in the same context to the same codeword. This encoding strategy, while relying heavily on context information for decoding, can provide robustness against channel noise by using fewer codewords with greater Hamming distance between them. For each encoding strategy, we fix the decoding function to be the minimum semantic error decoder in (43). We represent the actions of player 1 by $\mathcal{S}_1 = \{g_{ci}, g_{ca}\}$, where g_{ci} refers to the former, i.e., *context-independent*, encoding strategy, while g_{ca} is the latter, i.e., *context-aided*, encoding strategy.

The average semantic error is then evaluated from (10). Given $\Theta = \theta$, we denote the semantic error by $D_\theta(g_{ci}, p_B)$ when player 2 picks a bad distribution and player 1 picks a context-independent encoding function, $D_\theta(g_{ci}, p_G)$ when player 2 picks a good distribution but player 1 picks a context-independent encoding function, $D_\theta(g_{ca}, p_G)$ when player 2 picks a good distribution and player 1 picks a context-aided encoding function, and $D_\theta(g_{ca}, p_B)$ when player 2 picks a bad distribution and player 1 picks a context-aided encoding function. We require for $\theta \in \{a, h\}$,

$$D_\theta(g_{ca}, p_B) \geq D_\theta(g_{ci}, p_B) \geq D_\theta(g_{ci}, p_G) \geq D_\theta(g_{ca}, p_G) \tag{46}$$

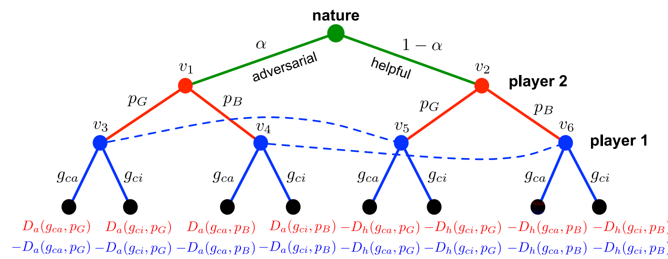


Fig. 3. Dynamic communication game model. Dashed lines represent the information sets of player 1 (encoder/decoder pair). Resulting payoffs are denoted by blue and red for players 1 and 2, respectively.

to ensure that it is better to use a context-independent encoding strategy against a bad distribution, and to use a context-aided encoding against a good distribution. If player 2 picks a good distribution, it is better to use a context-aided encoding strategy than a context-independent one. Both players have a finite number of actions. Lastly, we assume that both players remember their past actions and observations, i.e., the game has perfect recall [31, Sec. 11.1].

At the outset, the distinction between incomplete and imperfect information is for convenience, as the two problems are motivated by different scenarios. Incomplete information represents the scenarios in which the characteristics (objectives/types) of one or more players are unknown to the other players. Imperfect information represents scenarios in which the actions taken by one or more players cannot be observed by others. Harsanyi [11] showed that this distinction is rather artificial, as the former can always be transformed to the latter. To do so, one introduces a new player called the *nature*. The nature's actions consist of choosing a type for each player. Each player can only observe the type chosen for itself, but not the types chosen for others. Due to the fact that players cannot observe all of the actions taken by the nature, the game has imperfect information. This is now the standard technique for analyzing games with incomplete information [12], [39]. Accordingly, we also transform our game using Harsanyi transformation, by introducing another player, *nature*, that selects the type of player 2. Player 1 cannot observe the actions of nature. This game is illustrated in Fig. 3. Points connected by a dashed line, called an *information set*, cannot be distinguished by the current player. For instance $\{v_3, v_5\}$ is an information set for player 1 in Fig. 3, and player 1 cannot distinguish between states v_3 and v_5 while taking an action. Instead, player 1 forms *beliefs* about the current state of the game and updates them at each round after observing the actions taken by player 2.

As shown in Section VIII-C, Nash equilibrium for such games may lead to equilibrium strategies for which the actions taken off the equilibrium path are not rational. Instead, a refinement of Nash equilibrium, known as sequential equilibrium, is used for analyzing sequential games with imperfect information [40]. Every sequential equilibrium is also a Nash equilibrium. In addition, sequential equilibrium ensures that: i) at each information set, the current player takes the best, i.e., sequentially rational, strategy, ii) strategies are consistent with the beliefs the players hold.

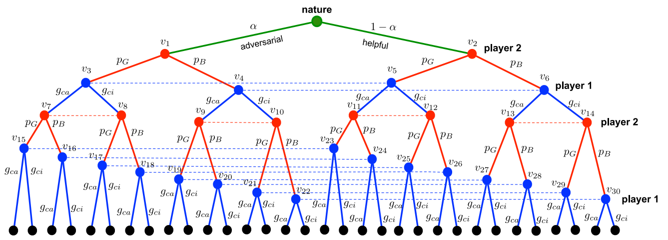


Fig. 4. Dynamic communication game model. Information sets of player 1 (encoder/decoder pair) and player 2 (agent) are represented by blue and red dashed lines, respectively.

We next formally state these necessary conditions, referring to [12], [31], and [40] for details. Let σ denote a behavioral strategy, which, at each information set, assigns a probability distribution to the set of available actions for the current player. It specifies the probability that the current player takes a specific action if the game is at one of the nodes belonging to the information set. We define σ_i as the behavioral strategy of player $i = 1, 2$ and $\sigma = (\sigma_1, \sigma_2)$. If player i takes an action at information set $\mathcal{I}$, $\sigma_i(k|\mathcal{I})$ is the probability that she takes action k.

We define a belief system μ to represent the beliefs the players holds about the current state of the game at each information set. As such, $\mu_{\mathcal{I}}(v)$ is the probability that the player taking an action at information set $\mathcal{I}$ believes the game is currently at node $v \in \mathcal{I}$, such that

$$\sum_{v\in\mathcal{I}} \mu_{\mathcal{I}}(v) = 1 \text{ for all } \mathcal{I}. \tag{47}$$

The strategy profile σ combined with a belief system μ is referred to as an *assessment* (σ, μ).

Suppose player i takes an action at information set $\mathcal{I}$. Define σ_k^t as the product of the probabilities governed by the behavior strategy σ from node k to node t. Let $\mathcal{T}_{\mathcal{I}}$ denote the set of all terminal nodes that can be reached from $\mathcal{I}$, and $u_i(t)$ denote the payoff user i receives at node $t \in \mathcal{T}_{\mathcal{I}}$. We then define the expected payoff for player i at $\mathcal{I}$ by

$$\mathbb{E}[u_i(\sigma|\mathcal{I},\mu)] = \sum_{v\in\mathcal{I}} \mu_{\mathcal{I}}(v) \sum_{t\in\mathcal{T}_{\mathcal{I}}} \sigma_v^t u_i(t), \tag{48}$$

where the expectation is based on the probabilities defined by the belief system μ as well as the behavior strategy σ.

Definition 1 (Sequential Rationality [12]): An assessment (σ, μ) is sequentially rational if, at every information set $\mathcal{I}$, the expected payoff for the current player i satisfies

$$\mathbb{E}[u_i(\sigma|\mathcal{I},\mu)] \geq \mathbb{E}\big[u_i\big(\sigma_i', \sigma_{-i}|\mathcal{I},\mu\big)\big] \tag{49}$$

for any alternative strategy σ_i' of player i, under the belief system μ.

One can prove the optimality of (49) by comparing, at each information set, only the one-step behavioral strategy of the current user, by fixing her strategy at all other information sets [41]. We let $P_\sigma(v)$ be the probability that a node v is reached in the game under strategy σ, which is given by the product of the behavioral strategies and nature's moves on the path from the root node to node v. Then, under strategy σ, an information set $\mathcal{I}$ is reached with probability

$$P_\sigma(\mathcal{I}) = \sum_{v\in\mathcal{I}} P_\sigma(v). \tag{50}$$

Definition 2 (Consistency) [12]: Denote by Σ_0 the set of all behavioral strategies σ that assigns a strictly positive probability to every action at each information set. Define Φ_0 as the set of all assessments (σ, μ) such that $\sigma \in \Sigma_0$, and μ is uniquely defined by the strategy σ from Bayes rule $\mu(v) = P^\sigma(v)/P^\sigma(\mathcal{I})$, where $\mathcal{I}$ is such that $v \in \mathcal{I}$. An assessment (σ, μ) is consistent whenever

$$(\sigma, \mu) = \lim_{n\to\infty} \left(\sigma^{(n)}, \mu^{(n)}\right) \tag{51}$$

for some sequence of assessments $(\sigma^{(n)}, \mu^{(n)}) \in \Phi_0$.

Definition 3 (Sequential Equilibrium) [12], [40]: An assessment (σ, μ) is a sequential equilibrium if it is consistent and sequentially rational.

Proposition 1: The dynamic semantic communication game has a sequential equilibrium.

This proposition immediately follows from the fact that every finite sequential game with imperfect information and perfect recall has a sequential equilibrium [40]. We note that Fig. 3 demonstrates a *signaling game*, in which the action chosen by a player *signals* its type to the other party [12]. Equilibrium structures in such games are often classified as: a *separating equilibrium* if different types of player 1 always chooses different actions, allowing player 2 to perfectly infer its type; a *pooling equilibrium* if player 1 always chooses the same action irrespective of its type, thus hiding its type from player 2; a *hybrid equilibrium* in which player 1 randomizes between its actions. It can be shown that the unique equilibrium for the game in Fig. 3 is a separating equilibrium in which the agent always chooses a bad distribution if adversarial, and a good distribution if helpful. The game in Fig. 3 allows each player to take a single action. Fig. 4 shows a larger game, where players again take actions one after the other, but they take two actions in total instead of one. Specifically, this larger game corresponds to the scenario in which the game in (3) is played twice. For space considerations, we only focus on the larger game in Fig. 4, and show that a similar separating nature exists in the actions of the agent. Accordingly, we assume that the payoffs received in Fig. 4 is the summation of the payoffs received at these two stages. For larger games, one can approximate sequential equilibrium via the quantal response equilibrium using [42], [43]. In the sequel, we demonstrate a sequential equilibrium for Fig. 4.

Proposition 2: Consider an assessment (σ, μ) with a behavioral strategy $\sigma = \{\sigma_1, \sigma_2\}$ such that

$$\sigma_1(g_{ca}|\mathcal{I}) = \begin{cases} 0 & \text{for } \mathcal{I} \in \{\{v_4, v_6\}, \{v_{16}, v_{24}\}, \{v_{18}, v_{26}\}, \{v_{20}, v_{28}\}, \{v_{22}, v_{30}\}\} \\ 1 & \text{for } \mathcal{I} \in \{\{v_3, v_5\}, \{v_{15}, v_{23}\}, \{v_{17}, v_{25}\}, \{v_{19}, v_{27}\}, \{v_{21}, v_{29}\}\} \end{cases} \tag{52}$$

$$\sigma_2(p_G|\mathcal{I}) = \begin{cases} 0 & \text{for } \mathcal{I} \in \{\{v_1\}, \{v_7, v_8\}, \{v_9, v_{10}\}\} \\ 1 & \text{for } \mathcal{I} \in \{\{v_2\}, \{v_{11}, v_{12}\}, \{v_{13}, v_{14}\}\} \end{cases} \tag{53}$$

where $\sigma_1(g_{ci}|\mathcal{I}) = 1 - \sigma_1(g_{ca}|\mathcal{I})$ and $\sigma_2(p_B|\mathcal{I}) = 1 - \sigma_2(p_G|\mathcal{I})$, and a belief system

$$\mu(v_i) = \begin{cases} 1 & \text{for} \quad i = 1, 2, 4, 5, 7, 10, 11, 14, 20, 22, 23, 25 \\ 0 & \text{for} \quad i = 3, 6, 8, 9, 12, 13, 15, 17, 28, 30 \\ \alpha & \text{for} \quad i = 16, 18, 19, 21 \\ 1-\alpha & \text{for} \quad i = 24, 26, 27, 29 \end{cases} \quad (54)$$

The assessment (σ, μ) is a sequential equilibrium.

Proof: The proof is provided in Appendix B. ■

The equilibrium in Proposition 2 suggests that adversarial and helpful agents always choose separate types of actions. An adversarial agent always chooses a bad distribution so that for each word, every context will be equally likely, whereas a helpful agent always chooses a good distribution. Against a bad distribution, the encoder always chooses a context-independent encoding, one which assigns each pair of words with different meanings to distinct codewords, irrespective of the belief it holds about the true nature of the agent. Against a good distribution, the encoder always chooses a context-aided encoding and leverage context information by assigning the words that have a small chance of occurring under the same context to the same codeword. We note, however, that this equilibrium structure depends on how the payoffs for the two parties are defined. For instance, if the payoffs were defined with respect to the average semantic error received at the last stage of the game instead of the sum of the errors at every round, then an adversarial player 2 might prefer to hide its nature by taking helpful actions until the last round.

VIII. Numerical Results

We consider a binary symmetric channel (BSC) with a crossover probability of ρ, i.e., the abstraction of binary communication over an additive white Gaussian (AWGN) channel with bit error probability ρ, between the encoder and the decoder. We focus on fixed length binary vectors of length n for the channel input and outputs, leading to the channel transition probability

$$p(\mathbf{Y} = \mathbf{y}|\mathbf{X} = \mathbf{x}) = \rho^{l(\mathbf{y},\mathbf{x})}(1-\rho)^{n-l(\mathbf{y},\mathbf{x})}, \quad (55)$$

where $l(\mathbf{y}, \mathbf{x})$ is the Hamming distance between $\mathbf{y}$ and $\mathbf{x}$. We consider a set of two contexts $\mathcal{Q} = \{q_1, q_2\}$. We select the words for our evaluations from a benchmark word set used in the semantic similarity literature [44], and let $p(W = w) = 1/|\mathcal{W}|$ for all $w \in \mathcal{W}$. We use the edge-based semantic similarity from Section II, illustrated in Fig. 1b. This is a corpus-independent similarity measure which allows us to avoid biasing our results in favor of a specific corpus.

A. Base Model With $|\mathcal{P}| = 1$

We initially study the structure of the encoding/decoding functions from Section VI by letting $|\mathcal{P}| = 1$ and fixing the conditional distribution between the contexts and words. We then study how Algorithms 1 and 2 perform in practice, by comparing them with an exhaustive search method that traverses over all the possible encoding/decoding functions to find the optimal assignments for the set of words,

$$\mathcal{W} = \{\text{car}, \text{automobile}, \text{bird}, \text{crane}\}, \quad (56)$$

where we set the remaining parameters as $n = 3$ and $\mathcal{X}^{(n)} = \mathcal{Y}^{(n)} = \{0,1\} \times \{0,1\} \times \{0,1\}$.

We consider two contexts such that,

$$q_1 = \text{things originating from non-living beings},$$

and

$$q_2 = \text{things originating from living beings}.$$

Accordingly, we let

$$p(Q = q_1|W = w) = \begin{cases} 1 & \text{if} \quad w = \text{car, automobile} \\ 0 & \text{if} \quad w = \text{bird} \\ 0.5 & \text{if} \quad w = \text{crane} \end{cases} \quad (57)$$

noting that $p(Q = q_2|W = w) = 1 - p(Q = q_1|W = w)$, and *crane* is meaningful in both contexts, where it takes the meaning of an *object that lifts and moves heavy objects* in context q_1, and a *large bird with a signature long neck* in context q_2 [29]. The same does not apply for the remaining words, however. For instance, *car* and *automobile* are irrelevant in context q_2, whereas *bird* is irrelevant in context q_1.

TABLE I
Average Semantic Error Comparisons

	$\lvert\mathcal{W}\rvert$	$\rho = 0.001$	$\rho = 0.01$	$\rho = 0.1$	$\rho = 0.2$
Algorithm 1	4	2.4626×10^{-6}	2.4479×10^{-4}	0.0444	0.2009
Algorithm 2	4	2.4649×10^{-6}	2.4479×10^{-4}	0.0230	0.0850
Exhaustive Search	4	2.4626×10^{-6}	2.4479×10^{-4}	0.0230	0.0850
Algorithm 2	20	0.002501	0.024784	0.224505	0.40856

We provide the semantic error values evaluated from Algorithms 1 and 2 in Table I, compared with exhaustive search results. The parameters used in the implementation of Algorithm 2 are $T_m = 10$, $T_f = 2.5 \times 10^{-8}$ and $N_{max} = 50000$. Table I indicates that Algorithm 2 performs close to exhaustive search. The performance of Algorithm 1, on the other hand, is inferior to both exhaustive search and Algorithm 2 as ρ grows larger. This result hints that Algorithm 2 is a better candidate than Algorithm 1 for evaluating the semantic error minimizing encoder/decoders for larger sets, for which exhaustive search is intractable. As such, we utilize Algorithm 2 in the sequel to determine the encoding and decoding policies for the following larger set of words,

$$\begin{aligned} \mathcal{W} = \{&\text{car, automobile, gem, jewel, coast, shore, stove, food,} \\ &\text{fruit, bird, forest, monk, brother, magician, crane,} \\ &\text{journey, voyage, furnace, noon, midday}\}, \end{aligned} \quad (58)$$

where we let $p(Q = q_1|W = w) = 1$ for $w \in${gem, jewel, coast, shore, stove, journey, voyage, furnace, noon, midday} and $p(Q = q_2|W = w) = 1$ for $w \in${food, fruit, forest, monk, brother, magician}. We select $n = 4$ and $\mathcal{X}^{(n)} = \mathcal{Y}^{(n)} = \{0,1\}^4$.

Semantic error values are as shown in Table I. Table II presents the corresponding encoding function, i.e., codeword assignments, for $\rho \in \{0.001, 0.01, 0.1, 0.2\}$. We observe from Table II that semantically closer words, such as *car* and *automobile*, are assigned to close codewords, as well as *gem*

TABLE II
ENCODING FUNCTION ($g(w)$) FROM ALGORITHM 2 FOR $w \in \mathcal{W}$

$g(w)$	$\rho = 0.001$	$\rho = 0.01$	$\rho = 0.1$	$\rho = 0.2$
$w =$ car	1100	0011	0000	1000
$w =$ automobile	1100	0011	0000	1000
$w =$ gem	0001	0101	1011	1111
$w =$ jewel	0001	0101	1011	1111
$w =$ coast	1010	1010	1110	1011
$w =$ shore	0111	0000	1111	1110
$w =$ stove	0100	1000	1100	0110
$w =$ food	1010	1111	0111	1100
$w =$ fruit	0011	1001	0001	1001
$w =$ bird	1011	1010	0011	1110

$g(w)$	$\rho = 0.001$	$\rho = 0.01$	$\rho = 0.1$	$\rho = 0.2$
$w =$ forest	0010	1101	0000	1000
$w =$ monk	0101	0010	1000	0111
$w =$ brother	1100	0000	1100	0011
$w =$ magician	1111	0100	1101	0001
$w =$ crane	1000	1100	0100	0110
$w =$ journey	0011	0001	1001	0010
$w =$ voyage	0010	0110	1101	0011
$w =$ furnace	0110	1001	1010	0000
$w =$ noon	1111	1111	0111	0101
$w =$ midday	1011	1111	0111	0101

TABLE III
NASH EQUILIBRIUM (NE) STRATEGIES FOR THE CODER AND THE AGENT WHEN $\alpha = 0.9$ AND $\alpha = 0.1$ FOR $|\mathcal{W}| = 4$

α=0.9	$s_1^* = g(w)$				$s_2^*(a)$	$s_2^*(h)$
	car	automobile	bird	crane		
NE 1	000	000	110	111	p_B	p_B
NE 2	000	000	101	111	p_B	p_B
NE 3	001	001	111	110	p_B	p_B
NE 4	110	110	000	001	p_B	p_B
NE 5	110	110	101	001	p_B	p_B
NE 6	111	111	001	000	p_B	p_B
NE 7	000	000	110	111	p_B	p_G
NE 8	000	000	101	111	p_B	p_G
NE 9	001	001	111	110	p_B	p_G
NE 10	110	110	000	001	p_B	p_G
NE 11	110	110	101	001	p_B	p_G
NE 12	111	111	001	000	p_B	p_G

α=0.1	$s_1^* = g(w)$				$s_2^*(a)$	$s_2^*(h)$
	car	automobile	bird	crane		
NE 1	000	000	000	111	p_B	p_G
NE 2	001	001	001	110	p_B	p_G
NE 3	110	110	110	001	p_B	p_G
NE 4	111	111	111	000	p_B	p_G

and *jewel* or *noon* and *midday*. Several semantically distant words are also assigned to the same codeword, such as *car* and *brother* when $\rho = 0.001$ or *magician* and *voyage* when $\rho = 0.1$. Since these words never occur in the same context, the decoder can use the context information to distinguish them. Upon comparing the completion time of Algorithm 2 for $|\mathcal{W}| = 20$ in Table I with exhaustive search when $\rho = 0.2$, we have observed that Algorithm 2 takes 1609 sec to complete, whereas exhaustive search could not terminate in a reasonable time frame.

Our results show that, to minimize error between the meanings of the recovered words, one should assign semantically closer words, i.e., words that are closer to each other in meaning, to closer codewords, those with a smaller Hamming distance. In that sense, words that are closer in meaning should be assigned to codewords that are confusable due to the noisy channel conditions.

B. Bayesian Game

We next implement the Bayesian game from Section IV by letting $\rho = 0.1$, $n = 3$, $\mathcal{X}^{(n)} = \{000, 001, 110, 101, 111\}$ and $\mathcal{Y}^{(n)} = \{0,1\}^3$. To reduce the dimensionality of the strategy spaces, for each given (pure) encoding strategy, we fix the decoder to the minimum error decoder described in (43). The coder now has to decide on the probability distribution only over the encoding functions, instead of both the encoder and the decoder, i.e., $s^*(g,h) = s^*(g)$. We evaluate the mixed strategy Bayesian Nash equilibrium by using the game theory tool Gambit [42].

We initially focus on the set of words from (56), and consider a context set $\mathcal{Q} = \{q_1, q_2\}$. For the agent, we define a strategy set $\mathcal{S}_2 = \{p_B, p_G\}$ as follows. Given $\Theta = \theta$, if the agent takes the action $s_2(\theta) = p_B$, the induced conditional probability between the words and contexts is

$$p(Q = q_1 | W = w, \Theta = \theta) = 0.5 \ \ \forall w \in \mathcal{W}, \tag{59}$$

whereas if the agent takes the action $s_2(\theta) = p_G$, the induced probability is

$$p(Q = q_1 | W = w, \Theta = \theta) = \begin{cases} 1 & \text{if} \quad w = \text{car, automobile} \\ 0 & \text{if} \quad w = \text{bird} \\ 0.5 & \text{if} \quad w = \text{crane} \end{cases} \tag{60}$$

Hence, under strategy p_B, contexts are uniformly distributed for each word, and the decoder cannot use context information to distinguish between any pair of words.

Table III demonstrates the evaluated equilibrium points for $\alpha = 0.9$ and $\alpha = 0.1$. In our results, we have observed 12 and 4 Nash equilibrium points for $\alpha = 0.9$ and $\alpha = 0.1$, respectively. Each point corresponds to a pure strategy Nash equilibrium. The agent's equilibrium strategy is $s_2^*(a)$ if adversarial and $s_2^*(h)$ if helpful. The coder's equilibrium strategy is $s_1^* = g(w)$ where $g(w)$ is the encoding function given in Table III. We observe that at equilibrium, synonyms *car* and *automobile* are assigned to the same codeword. When the agent is believed to be helpful, i.e., $\alpha = 0.1$, *bird* is assigned the same codeword with *car* and *automobile*. Since under the helpful agent's strategy *bird* never occurs in the same context with *car* and *automobile*, the decoder can use the context information to distinguish them. On the other hand, when the agent is believed to be adversarial, i.e., $\alpha = 0.9$, *car*, *bird*, and *crane* are all assigned to distinct codewords, since under an adversarial agent's strategy the decoder cannot use the context information to distinguish the words. In this case, the Hamming distance between *car* and *bird* is greater than the distance between *bird* and *crane*.

C. Dynamic Game

We implement the dynamic game in Fig. 4 from Section VII for the set of words from (56) with $\alpha = 0.2$, $\rho = 0.1$, $n = 3$, $\mathcal{X}^{(n)} = \{000, 001, 110, 101, 111\}$ and $\mathcal{Y}^{(n)} = \{0,1\}^3$. For the agent's actions, we let p_B and p_G as defined in (60). For

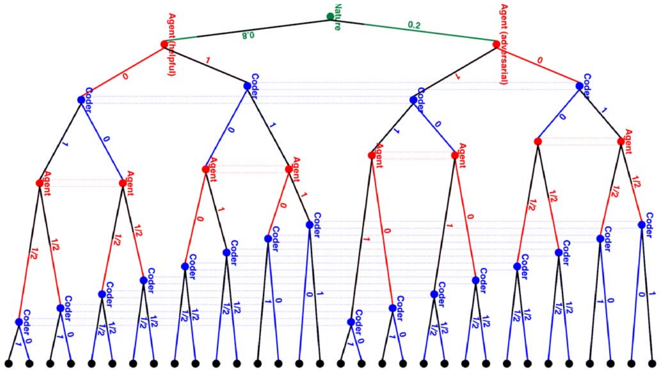


Fig. 5. Mixed strategy Nash equilibrium of the dynamic game.

the actions of the coder, we define the context-independent encoding function g_{ci} and the context-aided encoding function g_{ca} as,

$$g_{ci}(w) = \begin{cases} 000 & \text{if} \quad w = \{\text{car}, \text{automobile}\} \\ 110 & \text{if} \quad w = \{\text{bird}\} \\ 111 & \text{if} \quad w = \{\text{crane}\} \end{cases}$$

$$g_{ca}(w) = \begin{cases} 000 & \text{if} \quad w = \{\text{car}, \text{automobile}\} \\ 000 & \text{if} \quad w = \{\text{bird}\} \\ 111 & \text{if} \quad w = \{\text{crane}\} \end{cases}$$

and note that g_{ci} assigns each word with a different meaning to a different codeword, whereas in g_{ca}, we observe that the decoder cannot immediately distinguish *bird* from *car* or *automobile*, since they are assigned to the same codeword, and extra information is required to distinguish them. By using (43) to determine the corresponding decoding functions, we evaluate the average semantic error values in (46) using (10), from which we find that

$$D_\theta(g_{ca}, p_B) = 0.2260 > D_\theta(g_{ci}, p_B) = 0.0877$$
$$> D_\theta(g_{ci}, p_G) = 0.0433 > D_\theta(g_{ca}, p_G) = 0.023. \quad (61)$$

We then evaluate the mixed strategy Nash equilibrium of the dynamic game via a Linear Complementarity Program (LCP) using Gambit [34]. The equilibrium strategies are given in Fig. 5, where the edge labels indicate the probability that the corresponding action is played. The right (left) branch corresponds to the action g_{ca} (g_{ci}) for the coder and p_G (p_B) for the agent, respectively. It can be observed from the edges taken with probability 1/2 that the strategies off the equilibrium path are not necessarily rational. Unlike sequential equilibrium, these strategies are irrational if the player ever reaches to these states.

IX. Conclusion

We have considered the transmission of a source that carries a meaning through a noisy channel. An external entity can influence the decoder, whose true characteristic, e.g., friend or foe, is unknown to the communicating parties. We have formulated the semantic communication problem both as a static Bayesian game, for which we identified the Bayesian Nash equilibria, and as a dynamic game with imperfect information, for which we characterized a sequential equilibrium. Our results show that semantics-aware transmission schemes improve communication performance of intended meanings even in the presence of influencing agents with potentially adversarial actions. We note that in current communication systems such as LTE, content is strictly separated from the communication layer. The proposed semantic error measure is a new paradigm for designing future networks where communication may take place between humans and smart devices, such as smart home assistants or IoT devices. Unlike traditional communication systems, in these emerging networks, understanding the semantic content of the messages is not only performed by humans, but also by these smart devices. As a result, semantic inference is a direct part of the communication problem. Our goal in this paper has been to define a semantic error measure as a first step for integrating the semantic inference and physical communication problems. By integrating these two problems via the use of a semantic error measure, one may design messages carefully to save system resources and avoid transmitting irrelevant information. As this is an emerging area, there are several interesting future directions. One challenge is the complexity of the optimal encoding and decoding functions as the number of words increase. Accordingly, one may choose to replace words with sentences/phrases and measure the semantic similarity between sentences instead of words to scale up the system. Another interesting future direction is to take into account manipulation in the decoded message or false information, which is not considered in the current model. This includes considering more general scenarios with more capable agents, who can alter the content of decoded messages as well as influencing both the encoder and the decoder, with various malicious goals. Lastly, the estimation of the parameter α from past behavior patterns is an important future direction.

Appendix A

Proof of NP-Hardness of the Communication Problem

To prove the NP-hardness of the semantic communication problem, we need to show that there exists an NP-complete problem that is reducible to it in polynomial time. In the following, we show that there exists a polynomial-time transformation that reduces the quadratic assignment problem (QAP), which is known to be NP-complete [45], [46], to the semantic communication problem. The rationale behind this transformation is to show that, if one had a polynomial time algorithm to solve the semantic communication problem, then one could solve any QAP in polynomial time, by first transforming it to the semantic communication problem in polynomial time. This demonstrates that the semantic communication problem is at least as hard as QAP. Since QAP is known to be NP-complete, semantic communication problem is NP-hard.

Definition 4 (QAP [45]): Define a one-to-one function $\pi : \mathcal{W} \to \mathcal{L}$ where $\mathcal{L} = \{1, \ldots, |\mathcal{L}|\}$ with $|\mathcal{L}| \geq |\mathcal{W}|$, non-negative integer costs $\gamma(w, w')$ and distances $r(\pi(w), \pi(w'))$ for $w, w' \in \mathcal{W}$, and a bound $K \in \mathbb{Z}^+$. The QAP finds whether there exists a π such that

$$\sum_{w, w' \in \mathcal{W}, w \neq w'} \gamma\big(w, w'\big) r\big(\pi(w), \pi(w')\big) \leq K. \quad (62)$$

We follow the terminology from [45], and refer to [47] for generalizations. We let $p(\mathbf{Y}=\mathbf{y}|\mathbf{X}=g(w))$ denote $p(\mathbf{Y}=\mathbf{y}|\mathbf{X}=\mathbf{x})$ where $\mathbf{x}=g(w)$. Consider the decision form of the semantic communication problem from (40), i.e., given D_{max}, whether there exists $g:\mathcal{W}\rightarrow\mathcal{X}^{(n)}$ and $h:\mathcal{Y}^{(n)}\times\mathcal{Q}\rightarrow\mathcal{W}$ such that,

$$\sum_{\substack{w\in\mathcal{W},q\in\mathcal{Q},\mathbf{y}\in\mathcal{Y}^{(n)}:\\ h(\mathbf{y},q)\neq w}} p(\mathbf{Y}=\mathbf{y}|\mathbf{X}=g(w))p(Q=q,W=w) \times d(w,h(\mathbf{y},q))\leq D_{max}, \tag{63}$$

by using the fact that $d(w,h(\mathbf{y},q))=0$ whenever $w=h(\mathbf{y},q)$, and note that solving (63) is no harder than minimizing (40) [45]. By showing that (63) is NP-complete, we can prove that finding the encoding and decoding functions that minimize (40) is NP-hard.

Consider the following transformation that can be performed in polynomial time. Let $\mathcal{X}=\mathcal{Y}=\{0,1\}$. Consider some $n\in\mathbb{Z}^+$ such that $2^n\geq|\mathcal{L}|$, and a set $\mathcal{S}\subseteq\{0,1\}^n$ such that $|\mathcal{S}|=|\mathcal{L}|$. Let $\mathcal{X}^{(n)}=\mathcal{Y}^{(n)}=\mathcal{S}$. Set $\mathcal{Q}=\{q\}$ so that $|\mathcal{Q}|=1$. Define for all $\mathbf{y}\in\mathcal{Y}^{(n)}$ and $w\in\mathcal{W}$,

$$p(\mathbf{Y}=\mathbf{y}|\mathbf{X}=g(w))=1/|\mathcal{L}|, \tag{64}$$

and

$$p(W=w,Q=q)=1/|\mathcal{W}|. \tag{65}$$

Lastly, define for any $w\in\mathcal{W}$ and $\mathbf{y}\in\mathcal{Y}^{(n)}$,

$$d(w,h(\mathbf{y},q)) = \begin{cases} \frac{\gamma(w,h(\mathbf{y},q))r(\pi(w),\pi(h(\mathbf{y},q)))}{(|\mathcal{W}|^2+1)K\tau} & \text{if} \quad w\neq h(\mathbf{y},q),(w,\mathbf{y})\in\mathcal{F}\\ 0 \quad \text{if} \quad w=h(\mathbf{y},q)\text{OR}(w\neq h(\mathbf{y},q),(w,\mathbf{y})\in\mathcal{R})\\ 1 \quad \text{if} \quad w\neq h(\mathbf{y},q),(w,\mathbf{y})\notin\mathcal{F}\cup\mathcal{R}\end{cases} \tag{66}$$

where $\tau\geq1$ is a normalization factor $\tau=\max_{w,w'\in\mathcal{W},w\neq w'}\gamma(w,w')r(\pi(w),\pi(w'))$,

$$\begin{aligned}\mathcal{F}=\{(w,\mathbf{y}):\ &\text{i) } w\in\mathcal{W},\mathbf{y}\in\mathcal{S},\\ &\text{ii) } \mathbf{y}=g(w) \text{ for some } w\in\mathcal{W},\\ &\text{iii) } \forall w\in\mathcal{W}, h(g(w),q)=w,\\ &\text{iv) } \forall w'\in\mathcal{W}, w'\neq w\Rightarrow g(w)\neq g(w'),\\ &\text{v) } \forall\mathbf{y}'\in\mathcal{Y}^{(n)},\mathbf{y}\neq\mathbf{y}'\Rightarrow h(\mathbf{y},q)\neq h(\mathbf{y}',q)\Big\},\end{aligned} \tag{67}$$

and $\mathcal{R}=\{(w,\mathbf{y}):\text{ i) } w\in\mathcal{W},\mathbf{y}\in\mathcal{S},\text{ ii) } \mathbf{y}\neq g(w)$ for all $w\in\mathcal{W}\}$, noting that $\mathcal{F}\cap\mathcal{R}=\emptyset$. Lastly, let $D_{max}=\frac{1}{\tau|\mathcal{L}||\mathcal{W}|(|\mathcal{W}|^2+1)}$. The transformed problem can be stated as follows. Determine whether there exist $g:\mathcal{W}\rightarrow\mathcal{S}$ and $h:\mathcal{S}\rightarrow\mathcal{W}$ such that

$$\sum_{w\in\mathcal{W}}\sum_{\mathbf{y}\in\mathcal{S}:h(\mathbf{y},q)\neq w}\frac{1}{|\mathcal{L}||\mathcal{W}|}d(w,h(\mathbf{y},q)) \leq\frac{1}{|\mathcal{L}||\mathcal{W}|(|\mathcal{W}|^2+1)\tau}, \tag{68}$$

with $d(w,h(\mathbf{y},q))$ from (66). We next show that (62) has a solution if and only if (68) has one.

(⇒) For the *only if* part, suppose (62) has a solution. Set

$$g(w)=\pi(w) \tag{69}$$

for all $w\in\mathcal{W}$. For a given $\mathbf{y}\in\mathcal{S}$, let

$$h(\mathbf{y},q)=w \text{ if } \mathbf{y}=\pi(w) \text{ for some } w, \tag{70}$$

otherwise, set $h(\mathbf{y},q)=w$ to an arbitrary $w\in\mathcal{W}$. We note that for this assignment, $(w,\mathbf{y})\in\mathcal{F}\cup\mathcal{R}$ for all $(w,\mathbf{y})$ such that $w\in\mathcal{W}$, $\mathbf{y}\in\mathcal{S}$ and $w\neq h(\mathbf{y},q)$. The left hand side of (68) becomes

$$\begin{aligned}&\sum_{w\in\mathcal{W}}\sum_{\mathbf{y}\in\mathcal{S}:h(\mathbf{y},q)\neq w}\frac{1}{|\mathcal{L}||\mathcal{W}|}d(w,h(\mathbf{y},q))\\ &=\sum_{w\in\mathcal{W}}\sum_{\mathbf{y}:(w,\mathbf{y})\in\mathcal{F},h(\mathbf{y},q)\neq w}\frac{1}{|\mathcal{L}||\mathcal{W}|}\frac{1}{(|\mathcal{W}|^2+1)K\tau} \gamma(w,h(\mathbf{y},q))r(\pi(w),\pi(h(\mathbf{y},q))) \qquad (71)\\ &=\sum_{w\in\mathcal{W}}\sum_{w'\in\mathcal{W},w'\neq w}\frac{1}{|\mathcal{L}||\mathcal{W}|}\frac{1}{(|\mathcal{W}|^2+1)K\tau} \gamma(w,w')r(\pi(w),\pi(w'))\\ &\leq\frac{1}{|\mathcal{L}||\mathcal{W}|(|\mathcal{W}|^2+1)\tau}, \qquad (72)\end{aligned}$$

(71) holds since $(w,\mathbf{y})\in\mathcal{F}\cup\mathcal{R}$ for all $w\in\mathcal{W}$ and $\mathbf{y}\in\mathcal{S}$ for the assignments from (69) and (70); (71) holds since $d(w,h(\mathbf{y},q))=0$ whenever $\mathbf{y}\in\mathcal{R}$ as given in (66); (72) holds from (69), (70), the fact that π is one-to-one, and (62). Therefore, (68) has a solution.

(⇐) For the *if* part, suppose (68) has a solution. First, we show by contradiction that for any (g,h) that solve (68), $(w,\mathbf{y})\in\mathcal{F}\cup\mathcal{R}$ for all $w\in\mathcal{W}$, $\mathbf{y}\in\mathcal{S}$ with $w\neq h(\mathbf{y},q)$. Suppose $\exists(w',\mathbf{y}')$ with $w'\in\mathcal{W}$, $\mathbf{y}'\in\mathcal{S}$, and $w'\neq h(\mathbf{y}',q)$, but $(w',\mathbf{y}')\notin\mathcal{F}\cup\mathcal{R}$. Then, the left hand side of (68) is

$$\begin{aligned}&\sum_{w\in\mathcal{W}}\sum_{\mathbf{y}\in\mathcal{S}:h(\mathbf{y},q)\neq w}\frac{1}{|\mathcal{L}||\mathcal{W}|}d(w,h(\mathbf{y},q))\\ &\geq\frac{d(w',h(\mathbf{y}',q))}{|\mathcal{L}||\mathcal{W}|}\\ &\geq\frac{1}{|\mathcal{L}||\mathcal{W}|}\\ &>\frac{1}{|\mathcal{L}||\mathcal{W}|(|\mathcal{W}|^2+1)\tau}\end{aligned} \tag{73}$$

from (66) and the fact that $\tau\geq1$, which leads to a contradiction. Therefore, for any g and h that solve (68), one has $(w,\mathbf{y})\in\mathcal{F}\cup\mathcal{R}$ for all $w\in\mathcal{W}$, $\mathbf{y}\in\mathcal{S}$ with $w\neq h(\mathbf{y},q)$.

Next, we show by contradiction that g is one-to-one if it is a solution of (68). Suppose g is not one-to-one, i.e., there exist $w,w'\in\mathcal{W}$ such that $w\neq w'$ but $g(w)=g(w')$. Then, $\exists\,\mathbf{y}\in\mathcal{S}$ with $p(\mathbf{Y}=\mathbf{y}|\mathbf{X}=g(w))=p(\mathbf{Y}=\mathbf{y}|\mathbf{X}=g(w'))>0$ from (64), where $h(\mathbf{y},q)\neq w$ or $h(\mathbf{y},q)\neq w'$. Without loss of generality, let $h(\mathbf{y},q)\neq w'$. Then, $(w',\mathbf{y})$ leads to (73) and causes in a contradiction. Hence, g is one-to-one. Then, we can let $\forall w\in\mathcal{W}$,

$$\pi(w)=g(w), \tag{74}$$

and compute the left hand side of (62),

$$\sum_{w,w'\in\mathcal{W},w\neq w'}\gamma(w,w')r(\pi(w),\pi(w'))$$

$$= \sum_{w\in\mathcal{W}} \sum_{\mathbf{y}:(w,\mathbf{y})\in\mathcal{F},h(\mathbf{y},q)\neq w} \gamma(w,h(\mathbf{y},q))r(\pi(w),\pi(h(\mathbf{y},q))) \tag{75}$$

$$= \sum_{w\in\mathcal{W}} \sum_{\mathbf{y}:(w,\mathbf{y})\in\mathcal{F},h(\mathbf{y},q)\neq w} \big(|\mathcal{W}|^2+1\big)K\tau d(w,h(\mathbf{y},q)) \tag{76}$$

$$= \sum_{w\in\mathcal{W}} \sum_{\mathbf{y}\in\mathcal{S}:h(\mathbf{y},q)\neq w} \big(|\mathcal{W}|^2+1\big)K\tau d(w,h(\mathbf{y},q)) \leq K \tag{77}$$

where we use the fact that $(w,\mathbf{y}) \in \mathcal{F}\cup\mathcal{R}$ for all $w\in\mathcal{W}$, $\mathbf{y}\in\mathcal{S}$ with $w \neq h(\mathbf{y},q)$, and therefore $(w,g(w'))\in\mathcal{F}$ for all $w,w'\in\mathcal{W}$ with $w\neq h(g(w'),q)$, since by definition of $\mathcal{R}$, $(w,g(w'))\notin\mathcal{R}$ for all $w,w'\in\mathcal{W}$. Note that if $(w,g(w'))\in\mathcal{F}$ for some $w,w'\in\mathcal{W}$, then $h(g(w'),q)=w'$, and as a result, $(w,g(w'))\in\mathcal{F}$ for all $w,w'\in\mathcal{W}$ with $w\neq w'$. Observe also that if $(w,\mathbf{y})\in\mathcal{F}$ for some $w\in\mathcal{W}$, $\mathbf{y}\in\mathcal{S}$, then $\mathbf{y}=g(w')$ for some $w'\in\mathcal{W}$. Therefore, $(w,\mathbf{y})\in\mathcal{F}$ if and only if $\mathbf{y}=g(w')$ for some $w'\in\mathcal{W}$ such that $w\neq w'$, from which, (75) follows, since $\pi(w)=g(w)$ from (74). Then, (76) follows from (66), whereas (77) is from (68) and that $(w,\mathbf{y})\in\mathcal{F}\cup\mathcal{R}$ for all $w\in\mathcal{W}$, $\mathbf{y}\in\mathcal{S}$ with $w\neq h(\mathbf{y},q)$. Hence, (62) has a solution.

Lastly, we note that (63) is in NP since a nondeterministic algorithm need only guess a (g,h) pair and check in polynomial time whether (63) is satisfied. Hence, deciding whether there exist encoding/decoding functions whose average semantic error is below a threshold is NP-complete. It then follows that finding the semantic-error minimizing encoder/decoder is NP-hard.

Appendix B
Proof of Proposition 2

We first show that (σ,μ) is consistent and sequentially rational, then invoke Definition 3.

(Consistency) Define a behavioral strategy $\sigma^{(n)}$ such that,

$$\sigma_1^{(n)}(g_{ca}|\mathcal{I}) = \begin{cases} \frac{1}{n+1} & \text{for } \mathcal{I}\in\{\{v_4,v_6\},\{v_{16},v_{24}\},\\ & \quad\{v_{18},v_{26}\},\{v_{20},v_{28}\},\{v_{22},v_{30}\}\} \\ \frac{n}{n+1} & \text{for } \mathcal{I}\in\{\{v_3,v_5\},\{v_{15},v_{23}\},\\ & \quad\{v_{17},v_{25}\},\{v_{19},v_{27}\},\{v_{21},v_{29}\}\} \end{cases} \tag{78}$$

and

$$\sigma_2^{(n)}(p_G|\mathcal{I}) = \begin{cases} \frac{1}{n+1} & \text{for } \mathcal{I}\in\{\{v_1\},\{v_7,v_8\},\{v_9,v_{10}\}\} \\ \frac{n}{n+1} & \text{for } \mathcal{I}\in\{\{v_2\},\{v_{11},v_{12}\},\{v_{13},v_{14}\}\} \end{cases} \tag{79}$$

Since $0<\frac{1}{n+1}<1$ and $0<\frac{n}{n+1}<1$ for all $n\geq 1$, we have that $\sigma^{(n)}\in\Sigma_0$ where Σ_0 is as defined in Definition 2.

Let $\mu^{(n)} = \Big(\mu^{(n)}(v_1),\ldots,\mu^{(n)}(v_{30})\Big)$ denote the beliefs obtained from $\sigma^{(n)}$ via the Bayes rule from Definition 2. For the first round, beliefs of player 2 satisfy,

$$\mu^{(n)}(v_1) = \mu^{(n)}(v_2) = 1 \quad \forall n. \tag{80}$$

For the second round, beliefs of player 1 satisfy,

$$\begin{aligned}\mu^{(n)}(v_3) &= \frac{\alpha\sigma_2^{(n)}(p_G|\{v_1\})}{\alpha\sigma_2^{(n)}(p_G|\{v_1\})+(1-\alpha)\sigma_2^{(n)}(p_G|\{v_2\})} \\ &= \frac{\alpha}{\alpha+(1-\alpha)n} \xrightarrow{n\to\infty} 0 \end{aligned}\tag{81}$$

$$\begin{aligned}\mu^{(n)}(v_4) &= \frac{\alpha\Big(1-\sigma_2^{(n)}(p_G|\{v_1\})\Big)}{\alpha\Big(1-\sigma_2^{(n)}(p_G|\{v_1\})\Big)+(1-\alpha)\Big(1-\sigma_2^{(n)}(p_G|\{v_2\})\Big)} \\ &= \frac{\alpha}{\alpha+\frac{(1-\alpha)}{n}} \xrightarrow{n\to\infty} 1 \end{aligned}\tag{82}$$

where $\mu^{(n)}(v_5) = 1-\mu^{(n)}(v_3) \xrightarrow{n\to\infty} 1$ and $\mu^{(n)}(v_6) = 1-\mu^{(n)}(v_4) \xrightarrow{n\to\infty} 0$. For the third round, beliefs of player 2 satisfy,

$$\mu^{(n)}(v_7) = \mu^{(n)}(v_{11}) = \sigma_1^{(n)}(g_{ca}|\{v_3,v_5\}) = \frac{n}{n+1} \xrightarrow{n\to\infty} 1 \tag{83}$$

$$\mu^{(n)}(v_9) = \mu^{(n)}(v_{13}) = \sigma_1^{(n)}(g_{ca}|\{v_4,v_6\}) = \frac{1}{n+1} \xrightarrow{n\to\infty} 0 \tag{84}$$

where $\mu^{(n)}(v_8) = 1-\mu^{(n)}(v_7) \xrightarrow{n\to\infty} 0$ and $\mu^{(n)}(v_{10}) = 1-\mu^{(n)}(v_9) \xrightarrow{n\to\infty} 1$ and similarly $\mu^{(n)}(v_{12}) = 1-\mu^{(n)}(v_{11}) \xrightarrow{n\to\infty} 0$ and $\mu^{(n)}(v_{14}) = 1-\mu^{(n)}(v_{13}) \xrightarrow{n\to\infty} 1$. For the last round, beliefs of player 2 satisfy (85)-(86), as shown at the top of the next page, and that $\mu^{(n)}(v_{23}) = 1-\mu^{(n)}(v_{15}) \xrightarrow{n\to\infty} 1$. From a similar analysis, it follows for the remaining points that,

$$\mu^{(n)}(v_{16}) = 1-\mu^{(n)}(v_{24}) = \frac{\alpha n^2}{\alpha n^2+(1-\alpha)n^2} \xrightarrow{n\to\infty} \alpha \tag{87}$$

$$\mu^{(n)}(v_{17}) = 1-\mu^{(n)}(v_{25}) = \frac{\alpha}{\alpha+(1-\alpha)n^2} \xrightarrow{n\to\infty} 0 \tag{88}$$

$$\begin{aligned}\mu^{(n)}(v_{18}) &= \mu^{(n)}(v_{19}) = 1-\mu^{(n)}(v_{26}) = 1-\mu^{(n)}(v_{27}) \\ &= \frac{\alpha n}{\alpha n+(1-\alpha)n} \xrightarrow{n\to\infty} \alpha \end{aligned}\tag{89}$$

$$\mu^{(n)}(v_{20}) = 1-\mu^{(n)}(v_{28}) = \frac{\alpha n^2}{\alpha n^2+(1-\alpha)} \xrightarrow{n\to\infty} 1 \tag{90}$$

$$\mu^{(n)}(v_{21}) = 1-\mu^{(n)}(v_{29}) = \frac{\alpha n^2}{\alpha n^2+(1-\alpha)n^2} \xrightarrow{n\to\infty} \alpha \tag{91}$$

$$\mu^{(n)}(v_{22}) = 1-\mu^{(n)}(v_{30}) = \frac{\alpha n^3}{\alpha n^3+(1-\alpha)n} \xrightarrow{n\to\infty} 1 \tag{92}$$

Combining (78)-(79) with (80)-(92), we observe that $(\sigma^{(n)},\mu^{(n)})\to(\sigma,\mu)$ as $n\to\infty$ where (σ,μ) is as defined in (52)-(54). Hence, the assessment (σ,μ) is consistent.

(Sequential rationality): To show that the assessment (σ,μ) is sequentially rational, at each information set, we find the strategy that satisfies (49) for the current player by fixing his/her strategy at all other information sets, as well as the strategy of the other player.

$$\mu^{(n)}(v_{15}) = \frac{\alpha\sigma_2^{(n)}(p_G|\{v_1\})\sigma_1^{(n)}(g_{ca}|\{v_3,v_5\})\sigma_2^{(n)}(p_G|\{v_7,v_8\})}{\sigma_1^{(n)}(g_{ca}|\{v_3,v_5\})\Big(\alpha\sigma_2^{(n)}(p_G|\{v_1\})\sigma_2^{(n)}(p_G|\{v_7,v_8\}) + (1-\alpha)\sigma_2^{(n)}(p_G|\{v_2\})\sigma_2^{(n)}(p_G|\{v_{11},v_{12}\})\Big)} \quad (85)$$

$$= \frac{\alpha n}{\alpha n + (1-\alpha)n^3} \xrightarrow{n\to\infty} 0 \quad (86)$$

At information set $\mathcal{I} = \{v_1\}$, the expected payoff for player 2 becomes

$$\mathbb{E}[u_2(\sigma|\{v_1\},\mu)] = \begin{cases} D_a(g_{ca},p_G) + D_a(g_{ci},p_B) & \text{if } \sigma_2(p_G|\{v_1\}) = 1 \\ 2D_a(g_{ci},p_B) \quad \text{if} & \sigma_2(p_B|\{v_1\}) = 1 \end{cases} \quad (93)$$

and since $D_a(g_{ci},p_B) \geq D_a(g_{ca},p_G)$ from (46), we conclude that $\sigma_2(p_B|\{v_1\}) = 1$. At information set $\mathcal{I} = \{v_2\}$, the expected payoff for player 2 is

$$\mathbb{E}[u_2(\sigma|\{v_2\},\mu)] = \begin{cases} -2D_h(g_{ca},p_G) \quad \text{if} & \sigma_2(p_G|\{v_2\}) = 1 \\ -D_h(g_{ci},p_B) - D_h(g_{ca},p_G) & \text{if } \sigma_2(p_B|\{v_2\}) = 1 \end{cases} \quad (94)$$

and since $D_h(g_{ci},p_B) \geq D_h(g_{ca},p_G)$ from (46), we conclude that $\sigma_2(p_G|\{v_2\}) = 1$. At information set $\mathcal{I} = \{v_3,v_5\}$, player 1 has expected payoff

$$\mathbb{E}[u_1(\sigma|\{v_3,v_5\},\mu)] = \begin{cases} \mu(v_3)(-D_a(g_{ca},p_G) - D_a(g_{ci},p_B)) + (1-\mu(v_3)) \\ (-D_h(g_{ca},p_G) - D_h(g_{ca},p_G))\text{if}\sigma_1(g_{ca}|\{v_3,v_5\}) = 1 \\ \mu(v_3)(-D_a(g_{ci},p_G) - D_a(g_{ci},p_B)) + (1-\mu(v_3)) \\ (-D_h(g_{ci},p_G) - D_h(g_{ca},p_G))\text{if}\sigma_1(g_{ci}|\{v_3,v_5\}) = 1 \end{cases}$$

leading to $\sigma_1(g_{ca}|\{v_3,v_5\}) = 1$ as $D_\theta(g_{ci},p_G) \geq D_\theta(g_{ca},p_G)$ for $\theta \in \{a,h\}$ from (46). From a similar analysis, we find that $\sigma_1(g_{ci}|\{v_4,v_6\}) = 1$ using $D_\theta(g_{ca},p_B) \geq D_\theta(g_{ci},p_B)$.

At information set $\mathcal{I} = \{v_7,v_8\}$, player 2 has expected payoff

$$\mathbb{E}[u_2(\sigma|\{v_7,v_8\},\mu)] = \begin{cases} \mu(v_7)(D_a(g_{ca},p_G) + D_a(g_{ca},p_G)) + (1-\mu(v_7)) \\ (D_a(g_{ci},p_G) + D_a(g_{ca},p_G))\text{if}\sigma_2(p_G|\{v_7,v_8\}) = 1 \\ \mu(v_7)(D_a(g_{ca},p_G) + D_a(g_{ci},p_B)) + (1-\mu(v_7)) \\ (D_a(g_{ci},p_G) + D_a(g_{ci},p_B))\text{if}\sigma_2(p_B|\{v_7,v_8\}) = 1 \end{cases}$$

from which we find that $\sigma_2(p_B|\{v_7,v_8\}) = 1$ as $D_a(g_{ci},p_B) \geq D_a(g_{ca},p_G)$ from (46). From similar steps, we find that $\sigma_2(p_B|\{v_9,v_{10}\}) = 1$ and $\sigma_2(p_G|\{v_{11},v_{12}\}) = \sigma_2(p_G|\{v_{13},v_{14}\}) = 1$ using $D_\theta(g_{ci},p_B) \geq D_\theta(g_{ca},p_G)$ for $\theta \in \{a,h\}$.

At information set $\mathcal{I} = \{v_{15},v_{23}\}$, player 1 has expected payoff

$$\mathbb{E}[u_1(\sigma|\{v_{15},v_{23}\},\mu)] = \begin{cases} -\mu(v_{15})(D_a(g_{ca},p_G) + D_a(g_{ca},p_G)) - (1-\mu(v_{15})) \\ (D_h(g_{ca},p_G) + D_h(g_{ca},p_G))\text{if}\sigma_1(g_{ca}|\{v_{15},v_{23}\}) = 1 \\ -\mu(v_{15})(D_a(g_{ca},p_G) + D_a(g_{ci},p_G)) - (1-\mu(v_{15})) \\ (D_h(g_{ca},p_G) + D_h(g_{ci},p_G))\text{if}\sigma_1(g_{ci}|\{v_{15},v_{23}\}) = 1 \end{cases}$$

leading to $\sigma_1(g_{ca}|\{v_{15},v_{23}\}) = 1$ as $D_\theta(g_{ci},p_G) \geq D_\theta(g_{ca},p_G)$ from (46). From a similar analysis, one can find that, $\sigma_1(g_{ci}|\mathcal{I}) = 1$ for $\mathcal{I} \in \{\{v_{16},v_{24}\},\{v_{18},v_{26}\},\{v_{20},v_{28}\},\{v_{22},v_{30}\}\}$ and $\sigma_1(g_{ca}|\mathcal{I}) = 1$ for $\mathcal{I} \in \{\{v_{17},v_{25}\},\{v_{19},v_{27}\},\{v_{21},v_{29}\},\{v_{21},v_{29}\}\}$. Hence, (σ,μ) is sequentially rational using Definition 1. It then follows from Definition 3 that (σ,μ) is a sequential equilibrium.

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Başak Güler (S'13–M'18) received her B.Sc. degree in electrical and electronics engineering from Middle East Technical University (METU), Ankara, Turkey in 2009 and her M.Sc. and Ph.D. degrees in electrical engineering from Wireless Communications and Networking Laboratory, Pennsylvania State University, University Park, PA, in 2012 and 2017, respectively. She is currently a postdoctoral scholar at the Department of Electrical Engineering, University of Southern California. Her research interests include graph processing, distributed computation, source coding, social networks, and interference management in heterogeneous wireless networks.

Aylin Yener (S'91–M'01–SM'14–F'15) received the B.Sc. degree in electrical and electronics engineering and the B.Sc. degree in physics from Bogazici University, Istanbul, Turkey, and the M.S. and Ph.D. degrees in electrical and computer engineering from the Wireless Information Network Laboratory (WINLAB), Rutgers University, New Brunswick, NJ, USA. She is a Professor of Electrical Engineering at The Pennsylvania State University, University Park, PA, USA, since 2010, where she joined the faculty as an assistant professor in 2002, and was an associate professor 2006-2010. Since 2017, she is a Dean's Fellow in the College of Engineering at The Pennsylvania State University. She was a visiting professor of Electrical Engineering at Stanford University in 2016-2018 and a visiting associate professor in the same department in 2008-2009. Her current research interests are in caching systems, information security, green communications, and more generally in the fields of communication theory, information theory and network science. She received the NSF CAREER Award in 2003, the Best Paper Award in Communication Theory from the IEEE International Conference on Communications in 2010, the Penn State Engineering Alumni Society (PSEAS) Outstanding Research Award in 2010, the IEEE Marconi Prize Paper Award in 2014, the PSEAS Premier Research Award in 2014, and the Leonard A. Doggett Award for Outstanding Writing in Electrical Engineering at Penn State in 2014. She is a fellow of the IEEE, and a distinguished lecturer for the IEEE Communications Society and the IEEE Vehicular Technology Society.

Dr. Yener is a member of the Board of Governors of the IEEE Information Theory Society (2015-2020), where she was previously the Treasurer from 2012 to 2014. She served as the Student Committee Chair for the IEEE Information Theory Society from 2007 to 2011, and was the co-Founder of the Annual School of Information Theory in North America in 2008. She was a Technical (Co)-Chair for various symposia/tracks at the IEEE ICC, PIMRC, VTC, WCNC, and Asilomar in 2005, 2008-2014 and 2018. She served as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS from 2009 to 2012, an Editor and an Editorial Advisory Board Member for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS from 2001 to 2012, and a Guest Editor for the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY in 2011, and the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS in 2015. Currently, she serves on the Editorial Board of the IEEE TRANSACTIONS ON MOBILE COMPUTING and as a Senior Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS.

Ananthram Swami (F'08) is with the U.S. Army Research Laboratory (ARL) as the Army's ST (Senior Research Scientist) for Network Science. He is an ARL Fellow and Fellow of the IEEE. He has held positions with Unocal Corporation, the University of Southern California (USC), CS-3 and Malgudi Systems. He was a Statistical Consultant to the California Lottery, developed a MATLAB-based toolbox for non-Gaussian signal processing, and has held visiting faculty positions at INP, Toulouse, and Imperial College, London. He received the B.Tech. degree from IIT-Bombay, the M.S. degree from Rice University, and the Ph.D. degree from USC, all in Electrical Engineering. His research interests are in the broad area of network science with applications in composite tactical networks.